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On the Relationship Between Fertility and Public National Debt

by

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Abstract

Public debt and fertility are two issues of major concern in the current debate about economic policy, especially in countries with below-replacement-fertility and large debt (which appears further enlarged as a consequence of the recent world financial distress 2008-2009). In this paper we show that public debt is in general harmful for fertility, in that debt issuing almost ever crowds out fertility. The relationship is reversed only if debt is sufficiently low and the share of capital (labor) in the economy is sufficiently low (high). Hence, our analysis would recommend that developed, capital intensive economies (such as OECD countries) aiming at a fertility recovery should reduce national debt, while developing, labor intensive economies, aiming at reducing fertility, should increase (reduce) national debt only if they are debt virtuous (vicious).

J.E.L. classification: **D91, E62, H63, J13.**

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1. Introduction

Public debt and fertility are two issues of major concern in the current debate about economic policy. As regards the former issue, most OECD countries have experienced lasting budget deficits dating back to the mid-1970s and, as a consequence, rising debt to GDP ratios. In the EU's largest economy, Germany, public debt exploded in the years following the reunification; and in Asia's largest economy, Japan, the ratio of debt to GDP doubled during the 1990s. Remarkably, in Europe Italy, Greece and Belgium have the highest ratios of debt to GDP, namely larger than (or around) 100 per cent. Moreover, we note that public debt is further increasing, as a consequence of the financial crisis 2008-2009, in almost all advanced countries. As regards the second issue, the number of children per woman has fallen dramatically since the 1960s: however while in US such a number is around the replacement fertility rate, Japan and most European countries have a below-replacement fertility rate, in particular Italy.

Given the important size of public debt in many countries plagued by very low fertility rates (such as Italy), how and whether public debt affects fertility rates is an intriguing issue.

We note that while the literature has largely focused on the issue of the link between public debt and economic growth (e.g. Saint Paul, 1992; Josten, 2000; Bräuninger, 2005) little attention has been paid to the effect of the public debt on fertility[1]. This paper aims to fill this gap. The analysis is based on an OLG model (Samuelson, 1958; Diamond, 1965), extended in order to entail endogenous fertility motivated by a weak altruism of parents and in presence of constant public debt and lump-sum taxation, again strictly following Diamond (1965). We find that reducing debt policies, such as those advocated for such countries, may be either harmful or beneficial for a recovery of fertility, which is the other major policy target. In particular, the sign of the debt-reducing effect on fertility also depends, among other economic factors, on the size of the outstanding public debt: when such a debt is very high it is likely that its reduction favors a recovery of fertility. More precisely, we pick up the conditions under which debt-reducing policies may imply a fertility recovery: the latter occurs in the presence of a) high capital share in the economy (and any level of national debt); b) low capital share in the economy and a sufficiently

[1] For example, Ono (2003) investigates the relationship between longevity and social security policy in a model with public debt, but fertility rates are exogenous. For another study on debt and social security, see Gertler (1997), but again with exogenous fertility. Notable exceptions are, for example, the works by Lapan and Enders (1990) and Wildasin (1990), which analyze debt neutrality in presence of endogenous fertility, and Zhang (2003) and Spataro and Fanti (2011), in which the authors focus on the issue of debt optimality in presence of fertility choices.

high level of national debt. In all other cases debt reductions imply a decrease, rather than a recovery, of the fertility rates[2].

Interestingly, we show that the higher the cost of rearing children, the less likely is that a recovery of fertility is induced by debt reduction policy. This means that the debt tightening objectives being currently pursued by several developed countries can go hand in hand with the recovery of fertility provided that the child bearing costs are sufficiently low or reduced accordingly.

We also provide a rule of thumb for forecasting the effect of debt management on the rate of growth of population: if the latter is lower that the rate of interest, then its relationship with national debt is negative: if this is the case, then debt reductions (increases) surely boost (reduce) fertility.

Our results have straightforward policy implications for those countries plagued by both very high public debt levels and very low fertility rates: 1) indicating the cases in which a trade-off between the targets of debt reduction and fertility recovery may occur (i.e. with high capital share and/or high public debt); 2) indicating the interventions useful for avoiding such a trade-off (i.e. by accompanying debt reductions with reductions of children's costs).

Therefore, given the different economic parameters and the different size of the outstanding public debt in various countries, the effect of debt-reducing policy is an empirical matter. The paper is organized as follows: after laying out the model, in section 3 we present the results and in section 4 we conclude the paper.

2. The model set up

We adopt a standard OLG model (Diamond, 1965), extended with endogenous fertility[3], by assuming that life is divided into three periods (childhood, young adulthood, and old-age). In their childhood individuals do not make any decisions. Young adults are entailed with a well behaved utility function U defined over consumption in the second and third period of life ($c_{1,t}$, $c_{2,t+1}$) and on the number of children per adult (n_t), respectively. In words, in period t a representative agent born at time t-1, receives a salary w for her/his labour services (exogenously supplied) and decides how to split such an income over consumption in the same period, either for her/his own adulthood needs or on children rearing (we assume that each child costs a fixed amount of resources, q)[4]. Since we assume for simplicity that every single young adult can have children, the population at the steady state will be stationary or increasing if n is equal or bigger than 1 (thus n-1 is the long run growth rate of the population).

2.1. Firms

Each firm owns CRS production technology $F(K_t,L_t)$ which allows to transform physical capital K_t and labour L_t (which is equal to the young-age population N_t) into a consumption good Y_t. Under the hypothesis of perfect competitive markets, each firm hires capital and labour by remunerating them according to their marginal productivity. By defining $k=K/L$ the capital intensity, homogeneity of degree one of F yields $w_t = f(k_t) - f'(k_t)k_t$ and $r_t = f'(k_t)$ (in the case of absence

[2] Note that in an interesting paper by Zhao (2008), in which the link between public debt and fertility is addressed through a modified version of the Barro-Becker (1989) model, the relationship between public debt and fertility turns out to be unambiguously positive (via taxes for repaying public debt). Our model, instead, enriches the existing results and unveils the conditions under which such a relationship can be negative.

[3] We adopt a standard method for endogeneizing fertility in OLG models, following, for instance, Galor and Weil, (1996); Strulik, (1999) and (2003).

[4] This assumption departs from Strulik (1999) and (2003) who assumes the rearing cost as a fixed fraction of the wage. The assumption of a fixed children cost in terms of consumption goods rather than in terms of a fraction of income simplifies algebra without loss of generality.

of depreciation) or $r_t = f'(k_t) - 1$ (in the case of full depreciation), where low letters (apart from factor prices) indicate variables in per worker terms and f' indicates the derivative of f with respect to k_t. In particular, we adopt the usual Cobb-Douglas technology in intensive form: $y = Gk^h$ (where $G>0$ is a constant index of technology and h is the weight of capital in the production function as well as the distributive share of capital), and assume full depreciation of capital.

2.2. Government

Following Diamond (1965), we assume that the government at each date t issues a non-negative amount B_t of national debt and finances it by partly rolling it over and partly by levying lump sum taxes upon the young adults, according to the dynamic equation: $B_{t+1}=B_t(1+r_t) - \tau_{1,t}N_{t-1}$ (where $\tau_{1,t}$ is the lump sum tax) which, in per worker terms can be written as follows:

$$b_{t+1}n_t=b_t(1+r_t) - \tau_{1,t}; \quad [1]$$

moreover, again by following Diamond (1965), we assume that government pursues the constancy of debt in per worker terms, so that

$$\tau_{1,t}=b(1+r_t-n_t).$$

2.3. Individuals

The young adults face the following maximization problem:

$$\max U(c_{1,t}, c_{2,t+1}, n_t) = z_1 \log c_{1,t} + z_2 \log c_{2,t+1} + z_3 \log n_t$$

where $c_{1,t} = w_t - \tau_{1,t} - qn_t - s_t$ and $c_{2,t+1} = s_t(1 + r_{t+1})$.

Under our assumptions concerning public debt we get that, at the steady state:

$$s^* = z_2 q \frac{w - b(1+r)}{qv - bz_3} \quad [2]$$

$$n^* = z_3 \frac{w - b(1+r)}{qv - bz_3} \quad [3]$$

where $v = z_1 + z_2 + z_3$.

Note that by eq. [3], in this simple standard OLG frame the population growth depends positively on the wage, in line with a classical view à la Malthus. As regards the relationship between debt and fertility, it appears to depend crucially on the ratio between wages and return to capital, so that the endogenous determination of such a price ratio is needed. In the sequel we will investigate the steady state general equilibrium of this economy.

2.4. Steady state equilibrium

Given the market clearing equation $s_t N_{t-1} = K_{t+1} + B_{t+1}$ or, equivalently, $s_t = (k_{t+1} + b)n_t$ and assuming interior solutions for s, n, the long run per worker capital turns out to be:

$$k^* = q\frac{z_2}{z_3} - b\,. \qquad [4]$$

It is worth noting, from eq. [4], that at the equilibrium there is a complete "crowding out" effect of the public debt upon the stock of capital, that is, a one to one correspondence between them (such an effect is in line with Diamond (1965)).

Moreover, again by inspection of eq. [4] we can provide the following remark.

Remark 1: The long run per worker capital is inversely linked with the factors increasing the population growth and, thus, depends positively on the rearing cost q and negatively on the preferences for children z_3 and positively linked with the factor increasing accumulation, that is with the degree of patience z_2.

In other words, Remark 1 can be summarized as follows: i) the higher the preference for children the less saving will be accumulated for older age; ii) when, for given preferences for children, the cost of rearing them is lower, more children will be grown and less saving accumulated. Note that these results appear to be at all coherent with the empirical evidence.

Finally, we note that necessary and sufficient conditions for obtaining interior solutions ($s>0$ and $n>0$) and positive steady state of capital are i) $b \le q\frac{z_2}{z_3}$ and ii) $b < \frac{w}{1+r}$. However, under full depreciation of capital and Cobb-Douglas technology, it can be easily shown that $\frac{w}{1+r} = \frac{(1-h)(qz_2 - z_3 b)}{hz_3} > b$ if $b < b_{\max} = \frac{(1-h)qz_2}{z_3}$, such that the satisfaction of condition ii) is also sufficient for condition i) to hold: in other words, public debt must be sufficiently low, especially when rearing costs and the degree of patience are low and preference for children is high and share of capital h is high. For the sake of simplicity in the paper we will assume that this condition is always satisfied.

3. The effects of debt variations on fertility

Let us start from some preliminary results which provide a first insight into the relationship between debt and fertility; moreover they also link the shape of such a relation to the difference between the rate of growth of population and the interest rate:

Proposition 1: If $n \le 1+r$, then $\frac{dn}{db} < 0$; if $n > 1+r$ then $\frac{dn}{db} \gtrless 0$; finally, $\left.\frac{dn}{db}\right|_{b\max} < 0$.

Proof: See Appendix A.1.

In order to provide the economic intuition of the result, let us rewrite the $\frac{dn}{db}$ function as follows:

$$\frac{dn(w,r,b)}{db} = \underbrace{\frac{\partial n}{\partial b}}_{+/-} + \underbrace{\frac{\partial n}{\partial w}\frac{\partial w}{\partial r}\frac{dr}{\partial b}}_{-} + \underbrace{\frac{\partial n}{\partial r}\frac{dr}{db}}_{-} \qquad [5]$$

By looking at such an expression, we note that the ambiguity of the sign of the derivative dn/db stems from the direct (or partial) effect of public debt on fertility $\partial n/\partial b$ and precisely: if $n<1+r$

("underaccumulation case") public debt is a net tax for the individual: since children are a normal good (given our logarithmic preferences), the negative income effect of the tax increase always causes a reduction of fertility rates. On the other hand, symmetrically, if $n > 1 + r$ ("overaccumulation case"), then public debt behaves as a net subsidy and, as a consequence, given the above mentioned normality of children, fertility increases.

As regards the general equilibrium effects (that is the effects of public debt on prices) the signs are clear: since the number of children is positively (negatively) linked with wages (interest rate), which, in turn, are positively (negatively) linked with the capital intensity, then, given the capital equation [4], fertility is always reduced by public debt increases through changes in prices.

To sum up, from Proposition 1 it descends that that for economies in "underaccumulation" (*n<1+r*), the consequences of debt policies on fertility are univocal: the higher debt, the lower fertility. Therefore, to the extent that most economies are in "underaccumulation", as argued by many economists[5], our results suggest that debt-reducing policies may help a fertility recovery and conversely that the recent wave of public debt issues consequent to the financial distress 2008-2009, may push further down fertility rates in many advanced countries already strongly plagued by below-replacement fertility.

However, things are dramatically different in presence of "overaccumulation" (*n>1+r*), which we will address in the remainder of the paper.

However, since up to now we have provided conditions based on variables that are endogenous to our model, we now turn to explore the role of the parameters underlying such conditions, i.e. under or overaccumulation cases.

To start with, let us write the following lemma, which provides some conditions on debt and the capital share *h* ensuring that "overaccumulation" ("underaccumulation") occurs.

Lemma 1: i) $n \underset{<}{\overset{>}{-}} 1+r$ iff $b \underset{>}{\overset{<}{-}} b' = q\dfrac{z_2(1-h)-hv}{z_3(1-h)}$; ii) if $h > h' = \dfrac{z_2}{v+z_2} < \dfrac{1}{2}$, it turns out that $b' < 0 \le b$, such that $n < 1+r \quad \forall b \in [0, b_{\max}), z_1, z_2, z_3 > 0$;

Proof: As for point i) the proof is trivial by substituting for the expression of w, stemming from the equilibrium equation $w = f - f'k = \dfrac{(1-h)(qz_2 - z_3 b)}{hz_3}(1+r)$, *into eq. [3] and then solve the inequality* $n \underset{<}{\overset{>}{-}} 1+r$ *for b; as for point ii) the proof descends from observation of the expression for* b'. □

From Lemma 1 it descends that underaccumulation (*n<1+r*) can occur in either of the following cases: i) if the capital intensity of the economy is sufficiently low ($h < h'$) *and* the outstanding level of debt is higher than a threshold level (i.e. $b > b'$); ii) if the capital intensity of the economy is sufficiently high ($h > h'$).

In the light of the result above, we can provide the following Proposition, whose proof descends immediately from Lemma 1:

[5] As for the most recent empirical evidence on such issue, Abel et al. (1989) confirms that the underaccumulation situation holds for the U.S. economy and other six developed countries. However, it should also be noted that Anderson (1993) casts doubt on such conclusion as for the U.S., Canada and Great Britain.

Proposition 2: if $h > h' = \frac{z_2}{v + z_2}$, debt issuing reduces fertility, in that in this case (*n<1+r*).

Put it another way, when the capital intensity h is higher than a threshold level (h'), then the economy is in the overaccumulation case, such that any increase in the existing level of debt will be detrimental for fertility.

In the remainder of the paper we focus on the case $h < h'$, that is the situation in which the difference between the rate of growth of population and the interest rate is ambiguous, depending on the level of debt. If this is the case, recall that $n \gtrless 1 + r$ iff $b \lessgtr b' = q \frac{z_2(1-h) - hv}{z_3(1-h)} > 0$. Hence, we can study the function $\frac{dn}{db}$ with respect to the parameters of the model, focusing, in particular, on *h*. By substituting for the long run equilibrium values of $w = G(1-h)k^h$, $(1+r) = Ghk^{h-1}$, $k^* = q\frac{z_2}{z_3} - b$ into equation [3] and differentiating with respect to *b*, after some calculus we get the following equation for $\frac{dn}{db}$, whose sign depends on a quadratic form in debt:

$$\frac{dn}{db} = E\left[\sigma + \phi b + \gamma b^2\right] \qquad [6]$$

where $E = \frac{\left(\frac{qz_2 - bz_3}{b_3}\right)^a Gz_3^{\ 2}}{(qv - bz_3)^2 (qz_2 - bz_3)^2} > 0$, $\sigma = q^2 z_2\left[vh^2 - (2v + z_2)h + z_2\right]$, $\phi = \left[hv - (h-1)(h-2)z_2\right]qz_3 < 0$, $\gamma = z_3^{\ 2}(1-h) > 0$. Now, given that $\gamma > 0, \phi < 0$, it descends that in the $[0, b_{\max})$ interval the latter function can have either zero or one root if σ<0, or zero, one or two interior roots if σ>0. As for the case σ<0, since $\left.\frac{dn}{db}\right|_{b\max} < 0$, then it must be that *dn/db*<0 $\forall b \in [0, b_{\max})$ (because the latter function cannot change sign more than once and $\left.\frac{dn}{db}\right|_{b=0} = E\sigma < 0$). As for the case σ>0, since $\left.\frac{dn}{db}\right|_{b=0} = E\sigma > 0$ and $\left.\frac{dn}{db}\right|_{b\max} < 0$, we have one and no more than one positive root (let us call it b^n).

Finally, as for the sign of σ, we get that $\sigma \gtrless 0 \Leftrightarrow h \lessgtr h'' = 1 + \frac{z_2}{2v} - \sqrt{1 + \left(\frac{z_2}{2v}\right)^2} \in (0,1)$, (Note that $h' - h'' > 0$), and $b^n = q\frac{\Psi - \sqrt{(\Psi + 2vh)^2 - 4(1-h)^2 z_2^{\ 2}}}{2z_3(1-h)}$ where $\Psi = z_2(1-h)(2-h) - hv$.

Hence, from the above analysis we can provide the following Proposition:

Proposition 3: If $h'' < h < h'$, then increasing debt always reduces fertility. If $h < h''$, debt increases raise fertility until a threshold level of debt, $min(b', b^n)$, is reached, beyond which further increases reduce fertility.

Notice that we specified the condition $min(b', b^n)$ in that, in case $b' < b^n$, increases of debt beyond b' would generate underaccumulation ($n<1+r$), in which case, as we know from Lemma 1, debt increases beyond b' would produce fertility reductions, even before reaching the level of b^n.

We can summarize our results through Figure 1, where the loci b_{max}, b' and b^n are depicted in the h, b space. Note that both b' and b_{max} are decreasing functions of the capital intensity h and that, when $h=0$, all three loci take the value qz_2/z_3. Moreover, when $h < (=) h''$, then $b^n > (=) 0$ and $b' > 0$. As we argued in Proposition 2, when the capital intensity $h > h'$, then the economy is in the underaccumulation case ($n<1+r$), such that debt in this case increases are always detrimental for fertility. Following Proposition 3, the same conclusion occurs when $h \in [h'', h']$, although in this interval the economy can be either in the overaccumulation or in the underaccumulation case, depending on whether, according to Lemma 1, $b < b'$ or $b > b'$, respectively.

Finally, the conclusion can be reversed, that is, debt increases can boost fertility, provided that $h < h''$ and $b < b^n$.

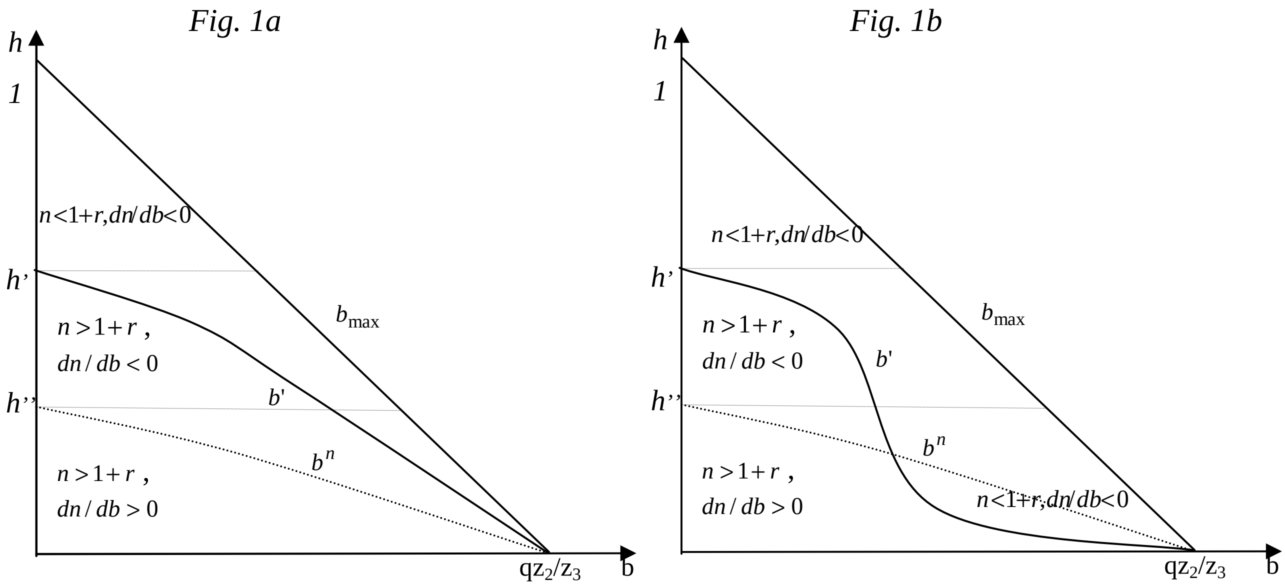

Figure 1: Parametric regions in the space *(h,b)*, where fertility is increasing or decreasing with debt. The qualitative shapes of the two figures are obtained through the following parameters configurations: *$G=1$, $z_1=1$, $z_2=z_3=0.5$, $q=0.6$ (Fig. 1a) and $q=0.5$ (Fig. 1b).*

To sum up, in the figure it emerges that debt issuing almost ever reduces fertility. The only exception is represented by the case in which both debt and capital shares are low.

Finally, the following interesting result, having straightforward policy implications, holds:

Corollary 1: The lower is the cost of rearing children (q) the more likely is a fertility stimulating effect of a debt reducing policy.

Proof : this straightforwardly follows by the derivatives $db_n/dq>0$ and $db'/dq>0$. Since h'' is not dependent on the cost of children q, then the Area in the bottom of the Figure 1 (below the b' and b^n loci, that is the parameter space in which the vicious couple debt reduction – fertility reduction occurs), is enlarged. □

The latter Corollary states that, if a government wishes to reduce its own level of debt and, at the same time, stimulate fertility rates, that is to end up outside the region at the bottom of Figure 1 (as it should be the case for countries largely indebted as well as less fertile such as Italy) it should also aim to a reduction of children's costs (for example, by enhancing the efficacy of publicly provided child services) in order to render more likely that their debt tightening policy also brings upon a recovery of fertility.

4. Conclusions

This paper extends the traditional OLG framework a là Diamond (1965) by allowing for endogenous fertility choices. Under this scenario we characterize the relationship between public debt and fertility, the latter representing important challenges for several advanced countries plagued both by large public debt and very low fertility rates, such as Italy. Therefore, for the latter countries it seems to be crucial, on the one hand, a reduction of public debt (for instance, as regards Italy, in order to comply with the Maastricht rules) and, on the other hand, a recovery of fertility rates (for instance, for avoiding concerns on the viability of PAYG pension systems). This appears even more cogent because of the recent increase in public debt issues set up to face the financial distress 2008-2009. The present analysis aims at unveiling the conditions whereby both objectives can be consistently pursued by national debt managing policies.

We point out that debt reductions can be either detrimental or beneficial for enhancing fertility, depending on economic factors such as technology, preferences and children costs and, interestingly, on the level of the outstanding public debt. In particular, we show that reducing debt is beneficial for fertility when the capital share of the economy and/or the outstanding level of debt are sufficiently high. Then a consequent policy implication is that, to the extent that developed economies (such as OECD countries) are relatively more capital intensive than developing economies, as assumed by some recent literature[6], our analysis would recommend that developed economies aiming at a fertility recovery should reduce national debt, while developing, labor intensive economies, aiming at reducing fertility, should increase (reduce) national debt only if they are debt virtuous (vicious).

Another interesting policy implication of our analysis is the following: the reduction of the cost of rearing children appears to be crucial for the current debt-tightening policies undertaken by several European countries to generate a recovery of population growth. In fact, countries such as Italy are more likely to be moving in the right direction provided that they accompany the current reduction of the public debt stock with policies designed to keep low (or, better, reduce) the costs of rearing children, so as to secure a recovery of fertility rates in the long run.

Finally, we also provide a rule of thumb for detecting characterizing the relationship between national debt and rate of growth of population: if the latter is lower than the rate of interest, then this relationship is negative. Therefore, the presence of "underaccumulation" is a sufficient condition for debt tightening policies to be beneficial for fertility. As a consequence, according to our results one should expect that the recent widespread policies of debt increases, as a remedy to the financial distress, will further lower the already below-replacement fertility rates of most developed countries.

[6] For instance Kraay and Raddatz (2007) assume a benchmark value of the average share of capital in value added in low-income countries of $h=0.5$, following in particular the Purdue University's Global Trade Analysis Project (2005) database (GTAP), "which is larger than thevalue of 0.36 typically used for developed countries (e.g., Kehoe and Perri, 2002)" (Kraay and Raddatz, p. 321).

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Appendix A: Proof of Proposition 1

Preliminarily, let us write the total derivative of the equilibrium demand for children n as follows:

$$\frac{dn(w,r,b)}{db}=\frac{\partial n}{\partial b}+\frac{\partial n}{\partial w}\frac{\partial w}{\partial r}\frac{dr}{db}+\frac{\partial n}{\partial r}\frac{dr}{db}=\frac{\partial n}{\partial b}-\frac{\partial n}{\partial w}k\frac{dr}{db}+\frac{\partial n}{\partial r}\frac{dr}{db} \qquad \text{[A.1]}$$

where we have exploited the equilibrium relationship $\partial w/\partial r=-k$ *, From individual's maximization problem, let us write as* $\Omega(n,b)=n-z_3\frac{w-\tau(b,\bar{n},r)}{qv}=0$ *the implicit function determining the economy's equilibrium value of n, where we have assumed that individuals do not take into account the effects of policy changes on the "aggregate population growth" rate (i.e.* $n=\bar{n}$ *into the government budget constraint, eq. [1]); however, relaxing such an assumption would change only slightly our results. Then, by using the implicit function theorem, we get the expression for* $\frac{\partial n}{\partial b}$ in

[A.1]*:* $\frac{\partial n}{\partial b}=-\frac{\Omega_b}{\Omega_n}=\frac{\frac{z_3}{qv}(n-1-r)}{1-\frac{z_3}{qv}b}\gtrless 0 \Leftrightarrow n\gtrless 1+r$ [7]*, since the denominator is positive under positive steady state capital. Moreover, we get that:* $\frac{\partial n}{\partial w}=-\frac{\Omega_w}{\Omega_n}=\frac{\frac{z_3}{qv}}{1-\frac{z_3}{qv}b}>0$ *and*

$\frac{\partial n}{\partial r}=-\frac{\Omega_r}{\Omega_n}=-\frac{\frac{z_3}{qv}b}{1-\frac{z_3}{qv}b}<0$. *Collecting terms we obtain:*

$$\frac{dn}{db}=\frac{\frac{z_3}{qv}}{1-\frac{z_3}{qv}b}\left[(n-1-r)-(k+b)\frac{dr}{db}\right]. \qquad \text{[A.2]}$$

Finally, by eq. [4] and by the properties of the Cobb-Douglas production function $f=Gk^h$*;*

$$\frac{dr}{db}=\frac{\partial r}{\partial k}\frac{\partial k}{\partial b}=-f''=-h(h-1)G\left[\frac{qz_2}{z_3}-b\right]^{h-2}>0. \qquad \text{[A.3]}$$

Hence, by eq. [A.2] when $n\le 1+r$*, dn/db<0. Furthermore, one gets that when* $b=b_{\max}=\frac{(1-h)qz_2}{z_3}$*,*

$$\left.\frac{dn}{db}\right|_{b_{\max}}=-\frac{z_3^{\,2}G\left(\frac{hz_2q}{z_3}\right)^h}{q^2z_2hv}<0.$$

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[7] One can show that this condition is equivalent to the following: $\frac{\partial n}{\partial b}>0 \Leftrightarrow n>1+r \Leftrightarrow \frac{w}{1+r}>\frac{vq}{z_3}$.