Tackling undeclared work[1]

Suggestions from a business cycle model with search frictions

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Abstract

The paper evaluates the relative effect of deterrence, prevention, curative and commitment policy measures on the size of undeclared work in a real business cycle model with moonlighting production, tax evasion and search frictions in the labor market. A numerical application of the model to the European economy shows that all these approaches reduce the undeclared share of output, but that deterrence and commitment policies also produce a negative effect on stationary employment. The curative approach produces the sharper fall in undeclared work while stimulating stationary output and employment.

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1 Introduction

The European debate on undeclared work (UDW) has devoted increasing attention to the identification of policy approaches and measures potentially able to tackle it, but has not yet produced convincing attempts to quantify and compare their relative effects on the undeclared share of output and the main labor market variables. This is somehow surprising, as the need for a thourough effectiveness evaluation, early stressed by the European Commission, is widely recognised.[3]

This shortcoming motivates our aim to start filling this evaluation gap by assessing the relative effects of deterrence, prevention, curative and commitment policy measures (discussed in the next section) on labor market variables and the size of UDW in a one-sector real business cycle model with search frictions in the labor market. The first type of policies are here represented by the sanctions applied to the firm that are caught employing UDW. The second approach is encapsulated in active labor market policies targeted at increasing the efficiency of declared work. The curative measures are exemplified by cuts in the labor tax rate and the commitment policies by measures able to influence the social stigma on UDW. We concentrate our attention on the long-run effects of the different types of policies, and hence on the stationary results of the model, but the dynamic behaviour of the maim variables when policy shocks hit the economy is also discussed, mainly focussing on the reactions at impact.

In order to model UDW and tax evasion, we adopt a moonlighting production scheme with three production factors: regular labor, underground labor, and a capital stock. This structure, that is in line with Busato et al. (2011) and Chiarini et al. (2009), hinges on the firms' ability to perform simultaneously recorded and unrecorded production activities.[4] In so doing, they use the same capital stock for both types of activities and to evade taxation on those they can hide from official recording. Although there is some evidence that this phenomenon is widely spread among countries (see e.g. World Bank 2000), the moonlighting scheme is particularly relevant in the context of more developed economies, where the use of UDW is mainly motivated by tax evasion. By calibrating the numerical version of the model on the European economy, we explore the quantitative effects of the four different types of policies and show that tighter repressive actions and commitment measures shrink the size of UDW, but at the cost of reducing output and increasing average unemployment. The curative approach and the preventive one do not generate this undesirable side-effect.

[3] The second European Commission Communication on undeclared work concluded that: "Piecemeal policy actions have been taken across Member States, but there is an apparent lack of evaluation of results" (European Commission 2007, p. 10).

[4] Several proposals to explore the underground economy within a dynamic framework charactherize instead the labor market as split into two separate sectors: one for regular employment, and one for the irregular/underground one (see, e.g., Castillo and Montoro 2010; Boeri and Garibaldi 2005).

The paper is organised as follows. In the next section we summarize the European debate on UDW and on the available policies to tackle it. In section 3 we describe the basic elements of the model economy. Section 4 illustrates its numerical version and discusses the calibration strategy. In section 5 we assess the effects of alternative policy approaches on the stationary ratio of undeclared income to total production and highlight the negative influence on the steady state values of employment generated by higher sanctions applied to the firm being caught employing UDW, and by greater social stigma. In the same section, we also analyze the impulse response functions produced by the model when it is subject to a shock on the labor tax rate and to a positive innovation affecting the productivity of declared work. Section 6 concludes.

2 The European debate on undeclared work

The debate on UDW is attracting increasing attention in the European Union. Although there exists no official definition of this phenomenon, the European Commision defines it as "productive activities that are lawful as regards their nature but are not declared to the public authorities, taking into account the differences in the regulatory systems between Member States" (Renooy et al, 2004, p. 95). In this context, undeclared differs from declared work only because it is concealed from the authorities in order to avoid taxes, social security contributions and other labor law constraints. Most important for our aims, this activities-based definition allows for the inclusion of forms of UDW that cannot be included in enterprise-based or jobs-based definitions, in particular "off the books, envelope wages", i.e., that part of the labor compensation that a formal employee in a registered company receives on an undeclared basis.

The European debate is developing along two main directions, On the one hand, the attempt is made at more accurately measuring the size of the phenomenon through direct methods.[5] On the other hand, there exists consensus on the importance of addressing the problem within a European dimension, in the framework of the European Employment Strategy. The interest for a cohordinated approach started in 1998, when the European Commission fostered a debate on the causes of UDW and on the best policies to tackle it, and invited the Member States to address the issue by envisaging the fight against UDW in their National employment strategies. In the following years, the need to strengthen cooperation among Member States led to the belief that fighting UDW is essential for the

[5] See the study on indirect measurement methods for undeclared work in the EU carried out by the Fondazione Giacomo Brodolini and GHK on behalf of the European Commission (VC/2008/0305).

achievement of the Lisbon employment goals. In particular, the 2003 European Employment Strategy Guidelines no. 9 on transforming UDW into regular work (Council resolution 2003/C 260/01) suggested the policy actions to be taken in order to tackle the problem, in the respect of national jurisdictions. These actions range from the introduction of measures able to facilitate compliance ("simplification of the business environment, removing disincentives and providing appropriate incentives in the tax and benefits system") to the punishment of non-compliance through "improved law enforcement and the application of sanctions". Measures aimed at enhancing fiscal morality and convincing on the fairness of paying taxes and social contributions were however disregarded.

Yet, after including UDW in the 2003-2005 Joint Work Programme, in 2007 the European Commission, in its Second Communication on UDW (*Stepping up the fight against undeclared work*) envisaged the fight against the undeclared economy as requiring a policy mix based on prevention measures, improved law enforcement and the application of sanctions, placing particular emphasis on the use of information campaigns aimed at increasing fiscal morality (European Commission 2007). This policy mix should, first of all, prevent workers from entering undeclared activities and, secondly, favour the transition form undeclared to declared work.

The European Commission further developed these issues in the 2008 – 2010 European Employment Strategy Guidelines (COM(2007) 803) and in the employment guidelines contained in part II of the Europe 2020 Integrated Guidelines (especially Guideline 7: Increasing labor market participation and reducing structural unemployment). This comprehensive approach must include several measures, including a legal and administrative framework favouring declared work (e.g., simplification of administrative procedures), a set of incentives/disincentives for the surfacing of the undeclared economy and strengthened inspections/penalties. Finally, the problem arising from the lack of coordinated mutual learning was also highlighted in a 2008 European Parliament resolution calling on Member States to step up the fight against UDW.

An overview report by the *European Foundation for the Improvement of Living and Working Condition* (Williams et al. 2008), based on the experience of five European countries (Belgium, Denmark, Italy, Poland and the UK), distinguishes between two main approaches to the fight against UDW. The first one, labeled the *deterrence approach*, aims at detecting and punishing non-compliance, or better at increasing the likelihood of detection and the costs of going undeclared, by introducing heavier penalties and sanctions (see, e.g., Grabiner 2000; Gramsick and Bursik 1990; Richardson and Sawyer 2001; Sandford 1999). The second, and recently emerging, one is labeled the *enabling compliance approach* (see, e.g., Slemrod 1992; Renooy et al. 2004; Williams 2006). This approach aims at encouraging tax and benefit compliance through the following three main sets of policies.[6]

1. *Prevention measures* discouraging people from engaging in UDW. These measure include simplification of administrative regulatory compliance, provision of new categories of contracts, business support and advice, tax incentives and facilitation of self-employment. Most important, prevention measures include the *active labor market policies* addressed to unemployed workers, especially those belonging to disadvantaged categories.

2. *Curative measures* aimed at favouring the transition of undeclared workers to declared work. These measures include tax and contribution amnesties, advisory services to facilitate businesses compliance and *tax measures reducing labor costs* or rewarding the purchasing of goods produced using declared work.

3. *Commitment measures* seek to encourage allegiance to tax morality (Andreoni et al. 1998; Cullis and Lewis 1997; Torgler 2003; Weigel et al. 1987; Wenze 2002). These measures include business advice and support, *awareness campaigns on the benefits of declared work* and tax fairness, and peer surveillance.

Williams et al. (2008) document that, first, the EU countries have heavily centered their policies on the deterrence approach, even though some attention has also been paid to the enabling compliance approach, focusing in particular on prevention measures. Commitment measures have instead remained rather unexplored. Secondly, the report claims that the available evidence is too limited to establish the relative effectiveness of the different approaches.

A second overview report by the *European Foundation for the Improvement of Living and Working Condition* (Williams and Renooy 2008) expands the previous analysis by examining the policies to tackle UDW realized in the 27 EU Member States and in Norway. This study confirms that the measures employed are different across European regions because different are the typologies of UDW, and that a one-size-fits-all solution is not applicable. It also suggests that, even though the deterrence type of policies have been prevalent, since the publication in 2003 of Guideline No.9 other types of measures belonging to the enabling approach have started to be more frequently employed. For example, in the countries where UDW is mostly widespread among dependent workers and is tightly intertwined with

[6] The two approaches have been variously named in the literature. See, e.g., Cullis and Lewis (1997); Murphy (2005); Ahmed and Braithwaite (2005).

the declared part of the economy (as in the case of "envelope wages") deterrence measures have been coupled with some prevention measures. Where UDW is instead more frequent among the self-employed the tendency has been to mostly employ enabling compliance policy measures. Even though the report tries to evaluate the effectiveness of the analyzed policy measures (when data are available), the authors have no difficulty in admitting that, in the face of a differentiated array of approaches and measures, a thorough assessment of their relative effectiveness remains unsatisfactory (Williams and Renooy, 2008, p. 1).

3 The model economy

In order to ground the assessment of the effectiveness of the four policy approaches on a simple model economy, (the analytical details of the model are available from the authors upon request) we consider only three (representative) agents: firms, households and a government. The government, which balances its budget (in expected terms) in each period, levies income taxes on regularly produced income flows, and labor taxes on regular labor services. Being subject to distortionary taxation, firms and households try to evade taxes by using UDW. The government may detect non-compliance and sanction the firms that are caught using this type of work.

In line with the recent literature, (see, among others, Shimer 2005) the economy is characterized by search frictions in the labor market which prevent some firms from filling all the vacancies they post (V_t) and some unemployed workers (U_t) in search of employment from finding advertised jobs. The number of realized matches is determined by the matching function $M_t = \eta(V_t)^{\xi}(U_t)^{1-\xi}$, where η is the efficiency of matching and ξ is the elasticity with respect to vacancies. The probability that a firm matches with a worker is $q_t = M_t/V_t$ and the probability that a worker matches with a firm is $p_t = M_t/U_t = \theta_t q_t$, where $\theta_t = V_t/U_t$ is the aggregate labor market tightness (from the viewpoint of the firm). The initial period unemployment is $U_t = 1 - N_t$ and employment evolves according to a dynamic equation specifying that existing matches may be severed for exogenous reason (with probability δ) at end of any given period and that new matches become productive in the next period: $N_t = (1-\delta)N_{t-1} + M_{t-1}$.

Household members, who are either employed or searching for a job, pool their resources in order to provide a mutual perfect insurance against differences in their labor income (Trigari 2009). The instantaneous utility function of the representative household is:

$$\begin{aligned} \mathcal{U}_t &= \log(C_t) - \mathcal{V}_t \\ \text{where} &: \mathcal{V}_t = B_0 \frac{(h_{M,t} + h_{U,t})^{1+\psi}}{1+\psi} + B_1 \frac{(h_{U,t})^{1+\psi}}{1+\psi} \end{aligned}$$

U_t is hence separable in consumption flow C_t (whose price is normalized to one) and in the overall disutility of labor V_t (where[7] $\psi, B_0, B_1 \geq 0$). $h_{M,t}$ and $h_{U,t}$ are the number of hours worked in the regular and in the underground sectors, respectively. The term $B_0 \frac{(h_{M,t}+h_{U,t})^{1+\psi}}{1+\psi}$ is the overall disutility of work, whereas $B_1 \frac{(h_{U,t})^{1+\psi}}{1+\psi}$ represents the disutility from undeclared hours worked, which we interpret as a subjective cost due to a "social stigma" (see, for instance: Benjamini and Maital 1985; Gordon 1989; Dhami and al-Nowaihi 2007) for working undeclared that can be influenced by commitment policies. The representative household maximizes the expected utility flows subject to the budget constraint:

$$\max_{\{C_t;K_{t+1}\}_0^\infty} E_0 \sum_{t=0}^{\infty} \beta^t \mathcal{U}_t \tag{1}$$

$$\text{s.t.} \quad : \quad C_t + K_{t+1} = (1-\tau_Y)\left(w_{M,t} N_t h_{M,t} + (r_t + \delta_K) K_t\right) + (1-N_t)\, b \tag{2}$$

$$+ w_U N_t h_{U,t} + \Pi^e + (1-\delta_K) K_t$$

where E is the expectation operator; $0 < \beta < 1$ is the subjective discount rate; K_{t+1} is next period capital stock; τ_Y is the distortionary rate on declared income flows; $w_{M,t}$ and $w_{U,t}$ are the declared and undeclared wages; r_t is the capital remuneration rate; δ_K is the capital stock depreciation rate; b is the (publicly financed) unemployment insurance, which encompasses all the resources devoted to income support (Hagedorn and Manovskii 2008); Π^e is the amount of expected profits accruing to the household . The budget constraint clarifies that the representative household may evade income taxes by reallocating labor services from declared to undeclared. The solution of this problem leads to a standard Euler equation for the intertemporal allocation of consumption.

The competitive (and representative) firm combines hired capital with declared and undeclared work according to a Constant Elasticity of Substitution production function in which both types of labor use the same capital stock:

$$Y_t = (K_t)^\alpha \left[(1-\omega)\left(\vartheta_t \cdot N_t h_{M,t}\right)^\rho + \omega \left(N_t h_{U,t}\right)^\rho\right]^{\frac{1-\alpha}{\rho}} \tag{3}$$

[7] Even though ψ could be different for $h_{M,t}$ and $h_{U,t}$, we adopt a common value.because of lacking empirical evidence on the supply elasticity of undeclared hours.

where Y_t is output, ϑ_t is a stochastic variable capturing idiosyncratic shocks hitting declared activities; $0 < \alpha < 1$, $0 < \omega < 1$ and $0 < \rho < 1$ are parameters. The firm tries to evade taxes on labor by using UDW. If the firm is detected evading (with probability p_D), it must pay the statutory tax rates on labor,τ_N, augmented by a surcharge factor, $s_D > 1$, and obtains profits $\pi_{D,t}$. If the firm is not detected evading (with probability $1 - p_D$), it obtains profits $\pi_{ND,t}$. These profits are defined as:

$$\pi_{D,t} = Y_t - (1+\tau_{N,t})w_{M,t}N_t h_{M,t} - (1+s_D\tau_{N,t})w_{U,t}N_t h_{U,t} - (r_t + \delta_K) K_t - \kappa V_t$$
$$\pi_{ND,t} = Y_t - (1+\tau_{N,t})w_{M,t}N_t h_{M,t} - w_{U,t}N_t h_{U,t} - (r_t + \delta_K) K_t - \kappa V_t$$

where κV_t is the costs of posting vacancies and κ is a parameter. The firm chooses vacancy posting to solve to problem:

$$F_t^V (N_t) = \max_{V_t, K_t, N_t} \left[\Pi_t^e + \beta E_t \frac{\mu_{t+1}}{\mu_t} F_{t+1}^V (N_{t+1}) \right] \tag{4}$$
$$s.t.\ (3), \quad N_t = (1-\delta)N_{t-1} + M_{t-1}$$

where F_t^V is the firm's value, $\Pi_t^e = p_D \pi_{D,t} + (1-p_D)\pi_{ND,t}$ are the expected profits and μ_t is the Lagrange multiplier of the household's problem. Using the first order condition with respect to V_t of problem (4) and the envelope theorem, we obtain the *job creating condition*, stating that the condition to post an additional vacancy at time t depends on the firm's discounted stream of future earnings and of savings on future vacancy posting (for the match that survive the exogenous job destruction):

$$\frac{\kappa}{q_t} = \beta E_t \frac{\mu_{t+1}}{\mu_t} \left[\begin{array}{c} \frac{\partial Y_{t+1}}{\partial N_{t+1}} - (1+\tau_{N,t+1}) w_{M,t+1} h_{M,t+1} \\ -(1+p_D s_D \tau_{N,t+1}) w_{U,t+1} h_{U,t+1} + (1-\delta) \frac{\kappa}{q_{t+1}} \end{array} \right]$$

The government budget constraint reads:

$$\begin{aligned} b(1-N_t) + G_t &= \tau_Y \left[w_{M,t} N_t h_{M,t} + (r_t + \delta_K) K_t \right] \\ &\quad + s_D p_D \tau_{N,t} w_{U,t} N_t h_{U,t} + \tau_{N,t} N_t h_{M,t} w_{M,t} \end{aligned} \tag{5}$$

where the right-hand side represents the expected government revenues and the left-hand side is composed of unemployment benefits, $b(1-N_t)$, and wasteful expenditure, G_t which is treated as an endogenous variable assuring that equation (5) holds in every t. The government balances its budget in expected terms since the tax revenues collected from firms' undeclared activities also depend on the probability of detection, p_D. Using the equation (5), together with the aggregate consumers' budget constraint and with firms' aggregate profits, it is possible to compute the aggregate resource constraint: $C_t + I_t + G_t = Y_t - \kappa V_t$.

In line with the existing literature, we assume that regular wages and hours are determined through an efficient Nash bargaining process. We assume instead that underground wages and hours are competitively determined. This choice is coherent with the moonlighting production scheme: while the agreement on regular labor services is subject to a normative set-up, the absence of a similar institutional framework for UDW leaves room for competitive forces to act.[8]

Households choose the supply of undeclared hours worked (for given $w_{U,t}$, $w_{M,t}$ and $h_{M,t}$) by maximizing utility (1) with respect to $h_{U,t}$ subject to their budget constraint (2). The resulting optimality condition equates the wage to the marginal rate of substitution between consumption and undeclared hours worked ($MRS_{U,t}$): $w_{U,t} = MRS_{U,t} \equiv \frac{\partial \mathcal{U}_t / \partial h_{U,t}}{\partial \mathcal{U}_t / \partial C_t}$. Firms choose the demand of $h_{U,t}$ (taking as given $w_{U,t}$, $w_{M,t}$ and $h_{M,t}$) by maximizing, with respect to undeclared hours worked, the value of the marginal worker employed in production (see equation (10) in the Appendix). The optimality condition $(1 + p_D s_D \tau_{N,t}) w_{U,t} = MP_{U,t}$ equates the wage (adjusted for for the tax rate on labor, the probability of detection and the surcharge) to the marginal productivity of undeclared hours of an additional worker: $MP_{U,t} \equiv \partial^2 Y_t / \partial (N_t) \, \partial h_{U,t}$. The optimality conditions for firms and households provide the equilibrium allocation rule for undeclared wage and hours worked, which equates the marginal rate of substitution to the adjusted marginal productivity of UDW:

$$MRS_{U,t} = \frac{1}{(1 + p_D s_D \tau_{N,t})} MP_{U,t} \tag{6}$$

As for declared wages and hours worked, firms and workers maximize the quantity $\Omega_t = (S_t^W)^d (S_t^F)^{1-d}$ with respect to $w_{M,t}$ and $h_{M,t}$ (taking as given $w_{U,t}$ and $h_{U,t}$). Ω_t is the Nash product of the rents arising from an existing match, where: S_t^F is the value of a match for a firm; S_t^W is the corresponding value for the worker (see the Appendix); (d) and $(1-d)$ are the exogenous bargaining power of workers/households and firms, respectively. From the optimalty condition for $h_{M,t}$ we obtain that the allocation rule for regular hours:

$$MRS_{M,t} \equiv \frac{\partial \mathcal{U}_t / \partial h_{M,t}}{\partial \mathcal{U}_t / \partial C_t} = \frac{(1 - \tau_Y)}{(1 + \tau_N)} MP_{M,t} \tag{7}$$

[8] In other papers with two distinct labor markets, both regular and underground wages are Nash bargained (see, e.g., Castillo and Montoro 2010; Boeri and Garibaldi 2005).

equates the marginal rate of substitution between consumption and regular hours ($MRS_{M,t}$) to the (tax adjusted) marginal productivity of regular hours of an additional worker, $MP_{M,t} = \partial^2 Y_t / \partial (N_t) \partial h_{M,t}$. The ratio between equations (6) and (7) univocally determines the ratio between regular and undeclared hours ($h_{R,t} = h_{U,t}/h_{M,t}$) as a function of the models' parameters:

$$h_{R,t}^{\rho-1} = \vartheta_t^{\rho} \left(\frac{1-\omega}{\omega} \right) \frac{(1-\tau_Y)(1+p_D s_D \tau_{N,t})}{(1+\tau_{N,t})} \left[1 + \frac{B_1}{B_0} \left(1 + h_{R,t}^{-1} \right)^{-\psi} \right] \tag{8}$$

Equation (8) explains that, in our model, the incentives to go undeclared are of three types: (i) technological reasons, represented by the parameters ω, ρ and ϑ_t that synthesize how regular and undeclared hours enter the production process; (ii) the fiscal incentive to hide labor, represented the tax rates (τ_Y and $\tau_{N,t}$) and the law and enforcement parameters (p_D and s_D); (iii) the (labor) supply side (dis)incentive of working undeclared, represented by the preference parameters (B_0, B_1 and ψ). When the productivity of regular hours ϑ_t^{ρ} and the labor tax rate $\tau_{N,t}$ are subject to stochastic shocks (as discussed in section 5.5), a short-run process of adjustment in $h_{R,t}$ takes place. The different channels affecting the ratio of the two types of hours worked allow us to compare several policy approaches aimed at tackling UDW. They also allow us to overcome the well known "tax evasion puzzle" (see, e.g., Allingham and Sandmo 1972 and Yitzhaki 1974), related to the inability of standard expected utility models to reconcile the relatively small size of tax evasion with the strong incentive produced by the empirically small values of p_D and s_D. In particular, the presence of the social stigma (B_1) and the higher relative productivity of regular worked hours (ϑ) reduce the incentive to evade taxes and help to obtain a realistic value for the share of undeclared income.

The declared wage turns out to be equal to (see the Appendix):

$$\begin{aligned} w_{M,t} &= \frac{(1-d)}{(1-\tau_Y) h_{M,t}} \left(C_t \mathcal{V}_t + b - w_{U,t} h_{U,t} \right) \\ &+ \frac{d}{(1+\tau_{N,t}) h_{M,t}} \left(\frac{\partial Y_t}{\partial N_t} - (1+p_D s_D \tau_{N,t}) w_{U,t} h_{U,t} + \mathcal{J}_t \right); \\ \text{where: } \mathcal{J}_t &= \frac{\kappa}{q_t} (1-\delta) E_t \left[1 - \left(1 - \frac{\theta_t q_t}{1-\delta} \right) \frac{(1+\tau_{N,t})}{(1+\tau_{N,t+1})} \right] \end{aligned} \tag{9}$$

The bargained regular wage is hence a weighted average of the players' reservation values. The worker's one is given by the sum of the overall disutility of work (evaluated in terms of consumption, $C_t \mathcal{V}_t$) plus the foregone flow benefit from unemployment (b), net of the wage income received when working undeclared. The firm's reservation value is given by the marginal revenue ($\partial Y_t / \partial N_t$), net of the undeclared labor cost, plus the expected net present value of remaining in the match ($\mathcal{J}_t$).

4 Baseline parameterization

In order to evaluate the model's performance and run some policy experiments, we calibrate the benchmark model on the 12 countries of the Euro Area considered in Christoffel et al. (2009)[9] and take from the same source the target for the stationary unemployment rate to $U^* = 9.1\%$ (which is the average 1984-2006 for the area). We also target $q^* = 0.71$ for the long-run vacancy filling rate, as in Christoffel et al. (2009). From steady state computations (starred variables indicate stationary values), we obtain the values of the job-finding rate and the labor market tightness: $p^* = 0.36$ and $\theta^* = 0.51$. Following Trigari (2009), we normalize to one the overall time devoted to work: $h_U^* + h_M^* = 1$ and from steady state computations we obtain $h_U^*/h_M^* = 0.57$, a value we cannot discuss in the absence of sufficient empirical information. We also choose the value of $b = 0.439$ to target a replacement rate $\frac{b}{w_M^* h_M^*} = 0.62$, which is close to the value 0.65 set by Christoffel et al. (2009). Given the focus on the moonlighting production scheme, the ratio of undeclared income over total production, S_U^*, plays a particularly important role in our analysis. For each of the 12 countries, we hence compute the ratio of its GDP to the whole area GDP for the period 2001-2011,[10] i.e., for country i we calculate: $gu_i = GDP_i / \left(\sum_{i=1}^{12} GDP_i \right)$. We then take from Schneider et al. (2010a and 2010b), for each country i, the share of the underground production over total output or the period 1996-2006. Finally, we compute the weighted average of the underground/output share for the whole EU12 area, by weighting the undeclared shares with the gu_i coefficients, obtaining for the same area the approximate value of 19% (19.13%) which we take as the value of the stationary ratio: $S_U^* = \frac{w_U^* h_U^* N^*}{Y^*}$. Table 1 summarizes the calibration targets:

Table 1: steady state target variables

$U^* = 0.091$	$q^* = 0.7$	$h_U^* + h_M^* = 1$	$S_U^* = 0.19$
$p^* = 0.36$	$\theta^* = 0.51$	$h_U^*/h_M^* = 0.57$	$\frac{b}{w_M^* h_M^*} = 0.62$

We fix most of the benchmark model's parameters in line with the literature (table 2):

Table 2: benchmark parametrization

Technology	$\alpha = 0.33$	$\omega = 0.53$	$\rho = 0.422$	$\delta_K = 0.025$	$\vartheta^* = 1.5$
	$\kappa = 0.3618$				
Preferences	$B_0 = 0.52$	$B_1 = 3$	$\psi = 2$	$\beta = 0.992$	
Tax structure	$\tau_Y = 0.128$	$\tau_N^* = 0.188$	$p_D = 0.04$	$s_D = 1.5$	
Labor market	$\xi = 0.5$	$\delta = 0.036$	$\eta = 0.5$	$d = 0.65$	$b = 0.425$

[9] That is: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal and Spain.

[10] Data from Eurostat, available at: http://epp.eurostat.ec.europa.eu/tgm/

As the data for the deterrence parameters, p_D and s_D, are typically lacking, except for few countries (see, for instance, the discussion in Torgler and Schneider 2007), we resort to the available evidence for Italy and the USA. Busato et al. (2011), use for the USA Joulfaian and Rider's (1998) values $s_D = 1.75$ and $p_D = 0.057$, while for the Italian economy Busato and Chiarini (2004) compute $s_D = 1.30$ and $p_D = 0.03$. Considering that Italy's and USA's values may be taken as the opposite extremes, the lack of better evidence forces us to take for the EU12 area the parameters values which are intermediate between those of Italy and the USA: $s_D = 1.5$ and $p_D = 0.04$. As for the tax rates, we calculate the (simple) averages for the EU12 countries from the EUROSTAT [14] report (Table 45 for tax rates on labor and Table 17 for income tax rates) and calibrate: $\tau_Y = 12.8\%$ and $\tau_N^* = 18.8\%$ for the stationary (long-run) values of the two tax rates. As for preferences and technology, the disutility parameters in the utility function $B_0 = 0.52$ and $B_1 = 3$, the elasticity parameter $\psi = 2$ (which is inside the range of empirically plausible values $\psi \in [0; 10]$) and two of the technology parameters, $\omega = 0.53$ and $\rho = 0.422$, are set so as to match the target $S_U^* = 19\%$. We choose for the discount rate the value $\beta = 0.992$ (corresponding to a annual rate of interest equal to 3.3%), which is commonly used in calibration exercises for the European economy. Finally, we set the relative productivity parameter $\vartheta^* = 1.5$ and conventionally adopt $\alpha = 0.33$ and $\delta_K = 0.025$. As for the labor market parameters, we set $\xi = 0.5$, coherently with the empirical evidence documented in Petrongolo and Pissarides (2001), $\eta = 0.5$ to match the target $q^* = 0.71$, and $\kappa = 0.3618$ and $d = 0.65$ to achieve the stationary employment target.[11] The quarterly separation rate is fixed at $\delta = 0.036$, as in Hobijn and Sahin (2007).

5 Policy experiments

In this section we study the effects of these four policy (the deterrence approach and the three measures listed above) approaches in the model presented in section 3. We do so by first using the benchmark parameterization and then comparing the long-run (i.e., steady state) equilibrium with that obtained under different values of the parameters encapsulating the policy measures previously discussed. More specifically, we assess the effects of the alternative policy approaches on the stationary ratio of undeclared income to total production, defined as $S_U^* = \frac{w_U^* h_U^* N^*}{Y^*}$, and highlight their influence on the steady state values of other relevant variables.

[11] This value is slightly higher than the conventional $d = 0.5$ often adopted for countries where the strength of the trade unions is lower than that experienced in the European economies considered in our exercise.

5.1 Deterrence measure: increase the penalty rate

We first study the effects of a progressive increase above the benchmark value of the penalty rate the state applies to firms caught underground, s_D. The results are shown in table 3:[12]

Table 3: effects of a deterrence measure (higher surcharge s_D)

	S_U^*	Y^*	U^*	N^*	h_M^*	w_M^*	θ^*
$s_D = 1.5$ (benchmark)	0.1904	1.5835	0.0918	0.9082	0.6350	1.0733	0.5077
$s_D = 1.7$	0.1901	1.5835	0.0918	0.9082	0.6353	1.0728	0.5070
$s_D = 1.8$	0.1899	1.5834	0.0919	0.9081	0.6356	1.0726	0.5067
$s_D = 1.9$	0.1898	1.5834	0.0919	0.9081	0.6357	1.0724	0.5064
$s_D = 2.0$	0.1896	1.5833	0.0919	0.9081	0.6359	1.0721	0.5061
$s_D = 3.0$	0.1879	1.5829	0.0922	0.9078	0.6377	1.0699	0.5029
$s_D = 4.5$	0.1854	1.5821	0.0926	0.9074	0.6403	1.0667	0.4982
$s_D = 9.0$	0.1783	1.5798	0.0937	0.9063	0.6481	1.0571	0.4848
$s_D = 18$	0.1654	1.5745	0.0960	0.9040	0.6628	1.0392	0.4600
Effect	↓	↓	↑	↓	↑	↓	↓

Note. The second line shows the benchmark values of selected steady state variables; the other lines contain the values of the same variables obtained by increasing s_D above the benchmark value.

The table shows that with higher penalty rates the stationary ratio of undeclared income over total production is lower. The fall is however very slow: s_D must almost triplicate in order to reduce S_U^* to 18.5%, and it must be raised by twelve times to bring the share of undeclared income to 16.5%.[13] Yet, the deterrence measure has a negative effect on stationary output and employment due to the increase in the overall labor cost linked to the greater amount of fines paid by firms following the rise in s_D.[14] Stationary unemployment hence monotonically increases with s_D. As a consequence, the greater number of searchers lowers the labor market tightness and, at the same time, the greater labor cost lowers firm's profit; the combined effects reduce the value of a match and hence of the bargained wage. Firms partly compensate the lower employment by increasing the declared intensive margin which becomes relatively cheaper than undeclared hours worked.

5.2 Prevention measure: favour the productivity of declared work

The second type of policy approach we aim to test is based on prevention policy measures, on active labor market policies (ALMP) in particular. To this aim, a possible line of analysis would be to ex-

[12] In this and in the other experiments discussed in the following sections the tables contain only selected values of the considered parameter. The general validity of our conclusions is however confirmed by the monotonicity of the results we obtain.

[13] Tax and deterrence have a small effect because the incentives to use UDW are also technological and preference-related, as discussed in Section 3.

[14] In Boeri and Garibaldi (2005) unemployment increases when the detection probability increases. However, since their labor market is split into two sectors, in the shadow segment job destruction increases and job creation falls, whereas regular employment increases.

plore the effects of changes in the efficiency of the matching technology, expressed by the parameter η. Nevertheless, this choice has a drawback. As we are modelling UDW in the context of a moonlighting production scheme $-$ which implies that the same worker performs both declared and undeclared work $-$ changes in the efficiency of the matching technology η can affect the value of the extensive margin, but (as shown by equation (8)) they leave unchanged the ratio of undeclared to regular worked hours $h_R^* = h_U^*/h_M^*$, which ultimately determines the relative size of undeclared production. The idiosyncratic productivity of regular work ϑ^*affects instead both h_R^* and the relative weight of underground activities on income (S_U^*), being at the same time sensitive to several types of factors influencing "daylight" working conditions, both in the long and in the short run. To our aims, it is then appropriate to perform a sensitivity analysis on ϑ^*, which in real word situations can be persistently affected by ALMP and by training in particular. The results of this experiment are shown in table 4.

Table 4: effects of a prevention measure (higher productivity ϑ^*)

	S_U^*	Y^*	U^*	N^*	h_M^*	w_M^*	θ^*
$\vartheta^* = 1.5$ (benchmark)	0.1904	1.5835	0.0918	0.9082	0.6350	1.0733	0.5077
$\vartheta^* = 1.6$	0.1864	1.6523	0.0879	0.9121	0.6423	1.1120	0.5576
$\vartheta^* = 1.7$	0.1826	1.7200	0.0846	0.9154	0.6491	1.1504	0.6072
$\vartheta^* = 1.8$	0.1790	1.7868	0.0816	0.9184	0.6555	1.1885	0.6565
$\vartheta^* = 1.9$	0.1757	1.8528	0.0790	0.9210	0.6615	1.2262	0.7054
$\vartheta^* = 2.0$	0.1725	1.9182	0.0766	0.9234	0.6672	1.2638	0.7541
$\vartheta^* = 2.5$	0.1590	2.2368	0.0674	0.9326	0.6916	1.4477	0.9934
$\vartheta^* = 3.0$	0.1482	2.5452	0.0610	0.9390	0.7112	1.6268	1.2273
Effect	$\downarrow\downarrow$	$\uparrow\uparrow$	$\downarrow\downarrow$	$\uparrow\uparrow$	$\uparrow$	$\uparrow$	$\uparrow\uparrow$

Note. The second column replicates the corresponding column of table 3; the other ones contain the values generated by the model when ϑ^* is progressively increased above the benchmark value.

Policies able to increase the efficiency of declared work are very effective in contracting the share of UDW: a 14% increase in the parameter ϑ^* reduces S_U^* below 18% and a 20% increase brings it down to 17.9%. If the efficiency of declared work is doubled, the undeclared share of income falls to below 15%. The policy measure is also able to increase stationary output, employment and the declared hours worked, and to curb unemployment. The main explanation for these results is related to the ability of an increase in ϑ^* to raise the overall productivity of the economy and to alter the allocation between the two types of labor services. An increase of the productivity of declared work induces an immediate increase in the overall productivity of employment $\partial Y^*/\partial N^*$,[15] which directly translates into an incentive to hire more workers and hence to expand output. At the same time, an increase in ϑ^* alters the optimal

[15] In order to verify that the overall productivity of employment incrreases with ϑ, consider the aggregate production function $Y = K^\alpha N^{1-\alpha} H^{\frac{1-\alpha}{\rho}}$, where $H = (1-\omega)(\vartheta h_M)^\rho + \omega (h_U)^\rho$. Employment productivity is then $\partial Y/\partial N = (1-\alpha)\frac{Y}{N}$ and its reaction to ϑ is: $\frac{\partial^2 Y}{\partial N \partial \vartheta} > 0$.

proportion between regular and undeclared labor services h_R^*: as regular worked hours are now relatively more productive, agents will opt for a reduction of this ratio, which drives the fall of the overall share S_U^*. Finally, the augmented productivity of N^* and the consequent fall in the number of searchers U^* raises the labor market tightness and increases the value of a match, as well as the bargained wage.

5.3 Curative measure: reduce the tax burden on labor

The reduction of labor costs obtained through tax cuts able to reduce the fiscal burden on employment has been frequently envisaged at the European level as a policy measure able to favor economic growth together with the contraction of UDW. In order to investigate on this intuition, we progressively decrease the stationary value of the labor tax rate, τ_N^*, below the benchmark calibration and consider the effects of this intervention in our model. The results of this test are summarized in table 5.

Table 5: effects of curative measures (lower labor tax rates τ_N^*)

	S_U^*	Y^*	U^*	N^*	h_M^*	w_M^*	θ^*
$\tau_N^* = 0.188$ (benchmark)	0.1904	1.5835	0.0918	0.9082	0.6350	1.0733	0.5077
$\tau_N^* = 0.17$	0.1901	1.5855	0.0912	0.9088	0.6388	1.0851	0.5143
$\tau_N^* = 0.16$	0.1899	1.5865	0.0909	0.9091	0.6410	1.0918	0.5180
$\tau_N^* = 0.15$	0.1896	1.5875	0.0906	0.9094	0.6432	1.0986	0.5217
$\tau_N^* = 0.13$	0.1892	1.5895	0.0901	0.9099	0.6476	1.1126	0.5290
$\tau_N^* = 0.11$	0.1887	1.5914	0.0895	0.9105	0.6520	1.1269	0.5363
$\tau_N^* = 0.1$	0.1885	1.5923	0.0892	0.9108	0.6543	1.1343	0.5400
$\tau_N^* = 0.05$	0.1873	1.5965	0.0879	0.9121	0.6660	1.1729	0.5581
Effect	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$

Note. The second line displays the benchmark values; the other lines show the values of the stationary variables corresponding to the tax cuts.

The ability of reductions in the tax burden on labor to shrink the area of UDW is confirmed by this experiment. Yet, even though the stationary ratio of undeclared income over total production monotonically decreases with the labor tax rate, the curative policy is only mildly effective: a tax rate four times lower than the benchmark value is able to reduce the share of the undeclared economy only to 18.7%. The policy measure is however able to increase the stationary output, employment and the declared hours worked, whereas stationary unemployment monotonically decreases with τ_N^*. These adjustments are triggered by the reduction in labor costs which has an impact on both the extensive and the intensive margin. As for the extensive margin, a fall in the overall cost of labor, $(1+\tau_N)w_M N h_M + (1+p_D s_D \tau_N) w_U N h_U$, fosters the hiring process and leads to an increase in employment N^*. The consequent reduction in the number of searchers raises the labor market tightness, and, analogously to the case of the ALMP, the bargained wage increases. As for the intensive margin, the lower tax rate modifies the relative allocation of regular and undeclared labor services, h_R^*: as the impact of a change in τ_N^* on the labor cost is stronger in the case of regular labor services than in the case of undeclared ones, the firms will choose a smaller level of h_R^*. As the expected cost of using undeclared labor, $(1+p_D s_D \tau_N) w_U N h_U$, must be weighted by the deterrence component $s_D p_D$, a fall in τ_N^* reduces the incentive to evade taxes and hence to use the undeclared type of labor input.

5.4 Commitment measure: increase the social stigma on undeclared work

We now assess the effects of commitment policies able to increase the subjective cost borne by the worker for the stigma placed by society on undeclared hours worked, as encapsulated in the parameter B_1. The results are shown in table 6.

Table 6: effects of a commitment measure (higher social stigma B_1)

	S_U^*	Y^*	U^*	N^*	h_M^*	w_M^*	θ^*
$B_1 = 3$ (benchmark)	0.1904	1.5835	0.0918	0.9082	0.6350	1.0733	0.5077
$B_1 = 3.3$	0.1888	1.5756	0.0923	0.9077	0.6422	1.0605	0.5009
$B_1 = 3.5$	0.1877	1.5707	0.0927	0.9073	0.6467	1.0527	0.4966
$B_1 = 4$	0.1853	1.5592	0.0935	0.9065	0.6568	1.0353	0.4868
$B_1 = 4.5$	0.1832	1.5487	0.0943	0.9057	0.6656	1.0203	0.4780
$B_1 = 5$	0.1814	1.5392	0.0950	0.9050	0.6734	1.0072	0.4700
$B_1 = 5.5$	0.1796	1.5304	0.0957	0.9043	0.6805	0.9955	0.4627
$B_1 = 6$	0.1781	1.5223	0.0964	0.9036	0.6868	0.9851	0.4559
$B_1 = 9$	0.1708	1.4832	0.0996	0.9004	0.7158	0.9388	0.4238
Effect	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$

Note. The second column contains the benchmark values of the variables considered for our policy experiments; the other columns display the values obtained when that parameter is increased above its benchmark value.

The stationary ratio of undeclared income over total production monotonically decreases when B_1 increases. The reaction of S_U^* is sharper than in the deterrence policy case: a value of B_1 twice as big as its benchmark calibration is associated with a share of undeclared income close to 17.8%. The increase in B_1 implies an increase in the utility cost of supplying undeclared hours worked, and this has the same effect as that of an increase in the real cost of using this input. As the supply of h_U^* is now more costly for workers, they would ask for a higher real wage w_U^* for the same amount of undeclared hours. On the other side of the market, firms would now find it more convenient to substitute undeclared services with regular ones. The final effect is a reduction of h_R^* which drives the fall of S_U^*. The results of this experiment also bring into light an important consideration which has been often disregarded in the current debate. The effects of an increase in the social stigma on UDW are similar to those produced by higher penalty rates: both measures bring about an undesired fall in the stationary levels of output and employment. This contractionary effect is related to the increase in the utility cost of undeclared hours. In a moonlighting scheme, both types of labor inputs are used in production and the rise in the cost of h_U^* induces a reduction in expected profits. The firms react to this increased burden by reducing their vacancy posting, which in its turn reduce the hiring of new workers. As in the previous cases, the lower labor market tightness has a negative impact on the bargained wage.

Summing up, our model suggests that, in the long-term, the deterrence and the prevention approaches are both able to reduce the ratio of undeclared income to total production, even though the latter approach is more effective. Yet, both types of policy must pay for this positive result the high price of generating lower output and employment, together with higher unemployment. Their ability to decrease the labor market tightness and declared wages cannot of course be assessed in general, being positive for firms and negative for workers. The curative approach based on the reduction of the tax burden on labor is mildly able to reduce UDW, generates no negative side-effects on output and employment, and favours a fall in unemployment which increases the labor market tightness and the bargained wage. The most effective approach is the preventive one, as it induces a sharp reaction in the share of UDW, increases stationary output, employment and declared hours worked, and raises the labor market tightness and the declared wage.

5.5 Dynamic effects of shocks on labor tax rate and on regular labor productivity

In order to deepen the analysis of the effects produced by policy measures centred on tax cuts and productivity enhancements, the ones that have no negative effects on employment, we discuss the dynamics taking place when the model is fed with a temporary (but highly persistent) negative shock $\varepsilon_{\tau_N,t}$ on the labor tax rate and a (highly persistent) positive shock $\varepsilon_{\vartheta,t}$ on the productivity of regular activities. The stochastic variables are modeled as AR(1) processes:[16] $\widehat{\tau_N}_t = \rho_{\tau_N}\widehat{\tau_N}_{t-1} + \varepsilon_{\tau_N,t}$, with $\rho_{\tau_N} = 0.9$ for the tax rate and $\widehat{\vartheta}_t = \rho_\vartheta\widehat{\vartheta}_{t-1} + \varepsilon_{\vartheta,t}$, with $\rho_\vartheta = 0.9$ for the productivity shock. For both shocks the standard deviation is set at 0.01. These shocks allow both the ratio $\widehat{h}_{R,t} = \widehat{h}_{U,t} - \widehat{h}_{M,t}$ and the ratio of undeclared income to total production, $\widehat{S}_{U,t}$, to fluctuate over the cycle. Our aim is to study the impact responses of the model's variables and the paths they follow on their way back to the steady state; these dynamics (obtained under the baseline calibration) are shown in figure 1.

The much higher impact responses of the model's variables to a positive shock on $\widehat{\vartheta}_t$ than to a negative shock on $\widehat{\tau_N}_t$ is striking. The difference is particularly evident as for output, employment, regular hours,

[16] Hatted variables indicate percentage deviations from steady state values.

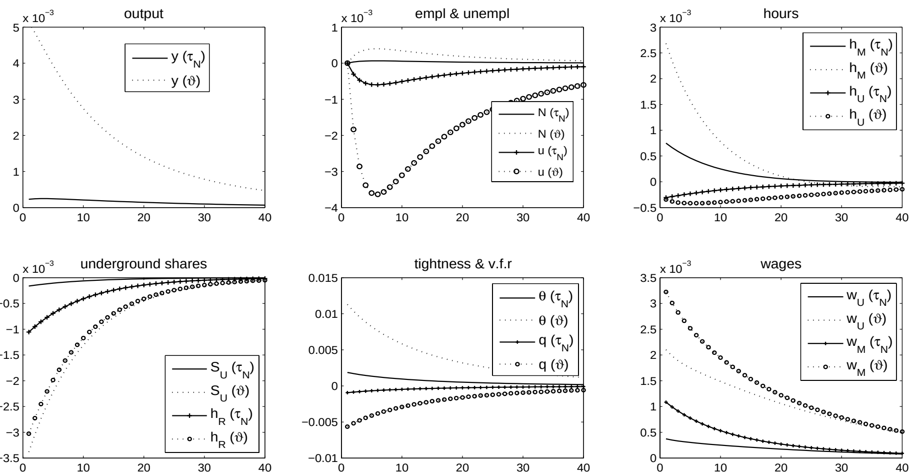

Figure 1: **Impulse response functions:** the figure shows the impulse response functions of the main endogenous variables to shocks on the tax rate on labor $(\widehat{\tau}_{N,t})$ and on the regular labor productivity $\left(\widehat{\vartheta}_t\right)$. In all the panels, solid lines show the variables' reaction to a shock on $\widehat{\tau}_{N,t}$, while dotted ones show the reaction of the same variables to a shock on $\widehat{\vartheta}_t$.

the ratio of undeclared income to total production, the labor market tightness and both wages. This shows that not only the long-run effects of productivity enhancements, but also the short-run ones are much more relevant than those of cuts in labor taxes.

Analogously to the discussion carried out in section 5.3, the shock on $\widehat{\tau_N}_t$ has a double nature: on the one side, it generates a reduction in the overall labor cost; on the other side, it induces a reallocation in favor of regular hours due to the reduced incentive to evade taxes and hence to use undeclared hours. The first effect induces an increase in employment, which is followed by an expansion of output. The reallocation effect has also an expansionary impact on $\widehat{Y}_t$, due to the decrease in $\widehat{h}_{R,t}$ as regular hours are more productive than their undeclared counterpart. The response of employment (extensive margin) is hence positive but lower and more persistent than the response of hours worked per employee (the intensive margin), as firms immediately adjust hours without sustaining search costs in order to expand output. This explains why the dynamic reaction of employment is hump-shaped. As searchers fall and increased profits induce firms to post more vacancies, the labor market tightness increases. In anticipation of a tighter labor market and higher profits, the value of an existing match increases, and a part of this higher value is captured by the worker, who can negotiate higher declared wages. Undeclared hours worked fall at impact and very slowly return to the steady state value. The same behavior is hence displayed by $\widehat{h}_{R,t}$. By inspecting the panels of figure 1, it can be noticed that the reactions of $\widehat{h}_{U,t}$ and $\widehat{w}_{U,t}$ are opposite in sign but equivalent in amplitude; at the same time, the rise in output is sharper than the increase in $\widehat{N}_t$ and, as a consequence, the overall productivity of employment rises (the percentage deviation of employment productivity from the stationary state is given by: $\widehat{Y}_t - \widehat{N}_t$). As a conclusion, the ratio of undeclared income to total production decreases at impact and smoothly returns to steady state.

The positive shock on $\widehat{\vartheta}_t$ also allows both $\widehat{h}_{R,t}$ and $\widehat{S}_{U,t}$ to fluctuate over the cycle. The form of the dynamic responses produced by this shock are similar to those of a tax shock: an increase in $\hat{\vartheta}_t$ raises the productivity of regular labor and this induces agents to expand the usage of this type of input. A reallocation effect favouring $\widehat{h}_{M,t}$ is also present. Higher productivity stimulates more production and employment; this generates the usual dynamics of labor market variables ($\widehat{V}_t$ and $\widehat{\theta}_t$ increase, whereas $\widehat{U}_t$ decreases), followed by and increase in the regular wage due to the combined effect of higher tightness and greater productivity. Analogously to the case of a tax shock, undeclared hours worked fall and the related wage rate increases. Whereas the fall in $\widehat{h}_{U,t}$ is due to the reallocation effect, the sharp increase in $\widehat{w}_{U,t}$ is driven by rise in the overall productivity of employment.

The analysis of the dynamic impact of changes in the tax rate and in the productivity parameter confirms, and strengthens, the conclusions on the long-run effects of these policy measures discussed in the previous sections. ALMP aimed at favouring the productivity of regular work services have a substantially stronger - and more persistent - impact than curative measures aimed at reducing the tax burden on labor. In addition to the consequences on the undeclared shares $\widehat{h}_{R,t}$ and $\widehat{S}_{U,t}$, changes in these two policy measures are also relevant for the short-run reaction of unemployment $\widehat{U}_t$: an even temporary improvement in ALMP has a substantially stronger (and favorable) impact response and a more persistent effect on the unemployment rate.

6 Conclusions

In this paper we made a first step towards the identification of the most effective policies to adopt in order to tackle UDW in the EU area. To the best of our knowledge, this is in fact the first attempt made in the literature to compare and quantify the relative effects produced on the main economic variables by the policies considered under the headings of "deterrence approach" and "enabling compliance approach". We show that measures aimed at repressing UDW by acting on controls and sanctions suffer from two main shortcomings: **i)** they are not very effective in reducing the undeclared share of income; **ii)** they produce negative effects on average output and employment. Commitment measures aimed at increasing the social stigma on UDW have the same side-effect, but are more effective in tackling the undeclared phenomenon. Tax measures reducing labor costs, as well as prevention measures aimed at increasing the productivity of the declared labor input, are able to reduce both UDW and average unemployment. Yet, both the long-run and the short-run effects of productivity enhancements are much greater than those of cuts in labor taxes.

The general intuition for the differentiated consequences of the four policy approaches relies upon the different channels through which they propagate their effects in the economy and on the different sets of economic incentives they affect. Deterrence and commitment policies reduce the incentives to use UDW mainly by increasing the costs associated to this particular input (real wage costs in the case of deterrence and utility costs in the case of commitment). However, in an economy where the two types of labor services are jointly used to carry out production, this increase in the overall cost of production negatively affect output and employment.

Policies based on tax reductions and on ALMP act *via* different routes. In the case of "curative" measures based on reductions in the tax rate on labor, the primary impact is still on the overall labor cost, but its direction is opposite to that of deterrence measures. The reduction in the tax burden can achieve two results at the same time. On the one hand, it reduces firms' incentive of evading taxes, which is tightly related to their convenience of using UDW in production; on the other hand, it stimulates production and employment via the standard cost-based mechanism: a reduction in firms' overall cost of employment stimulates the hiring of more workers and hence fosters economic activity. Nevertheless, the size of these desirable effects remains limited because the tax reduction acts through indirect cost channels. The mechanism through which ALMP impact UDW and the economy's activity level is different and more direct. These measures have a favorable impact on production and employment because they immediately make the economy more productive. Furthermore, by directly increasing the productivity of regular work, enhanced ALMP also make UDW less attractive as a production input.

When more generally interpreted, these results suggest that the fight against UDW should be primarily centred on education and ALMP, training in particular. It should however be noted that the deterrence approach, on the one side, and the enabling compliance approach, on the other side, are not equivalent in terms of their implementation costs. Just to provide an evocative example, implementing an improved system of ALMP may be more costly than adopting a more severe set of deterrence policy measures. For this reason, we reckon that the next step of our research must explicitly introduce these costs into the model economy and address the issue of how to carry out a thorough welfare analysis.

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Appendix: Nash bargaining

Denote with $S_t^F = S_t^J - S_t^V$ the firms' surplus from an existing employment relationship. S_t^J and S_t^V are the value for a firm of a filled and of an open vacancy:

$$\begin{aligned} S_t^J &= \frac{\partial Y_t}{\partial N_t} - (1+\tau_{N,t})w_{M,t}h_{M,t} - (1+p_D s_D \tau_{N,t})w_{U,t}h_{U,t} \\ &\quad +\beta E_t \frac{\mu_{t+1}}{\mu_t}\left[\delta S_{t+1}^V + (1-\delta)S_{t+1}^J\right] \end{aligned} \tag{A.1}$$

and

$$S_t^V = -\kappa + \beta E_t \frac{\mu_{t+1}}{\mu_t}\left[q_t S_{t+1}^J + (1-q_t)\, S_{t+1}^V\right] \tag{A.2}$$

According to equation (A.1) the value of a filled vacancy is given by current profits plus the continuation value. The latter is equal to S_{t+1}^J if the match in not destroyed and to S_{t+1}^V if a separation occurs. According to equation (A.2) the value of an open vacancy is given by the cost of the same vacancy plus the expected value of filling, or not filling, it in the next period. These values are multiplied by probabilities q_t and $1-q_t$ respectively, as a successful match becomes productive in the next period. As firms are owned by households, the firms' discount factor includes the relative change in the households' Lagrange multipliers.

Denote with $S_t^W = S_t^M - S_t^N$ the workers' surplus from an existing match. S_t^M and S_t^N, are the values the worker enjoys when being matched or unemployed:

$$\begin{aligned} S_t^M &= (1-\tau_{Y,t})\, w_{M,t}h_{M,t} + w_{U,t}h_{U,t} - \frac{B_0(h_{M,t}+h_{U,t})^{1+\psi}}{(1+\psi)\,\mu_t} \\ &\quad -\frac{B_1(h_{U,t})^{1+\psi}}{(1+\psi)\,\mu_t} + \beta E_t \frac{\mu_{t+1}}{\mu_t}\left\{(1-\delta)S_{t+1}^M + \delta S_{t+1}^N\right\} \end{aligned} \tag{A.3}$$

and

$$S_t^N = b + \beta E_t \frac{\mu_{t+1}}{\mu_t}\left[\theta_t q_t S_{t+1}^M + (1-\theta_t q_t)\, S_{t+1}^N\right] \tag{A.4}$$

Equation (A.3) shows that the value of being employed is equal to the wage income net of labor disutility. The following period the worker can still be employed in the firm (with probability $1-\delta$) and enjoy the value S_{t+1}^{M}, or be unemployed and hence in search because a separation occurred, obtaining S_{t+1}^{N}. Equation (A.4) has a similar interpretation: if the worker is matched with a firm at time t, with probability $p_t = \theta_t q_t$, he must wait a period to be actually employed, so to obtain S_{t+1}^{M}. If he remains unemployed, with probability $1 - p_t$, he obtains S_{t+1}^{N}.

The first order conditions with respect to $w_{M,t}$ and $h_{M,t}$ of the Nash bargaining problem

$$\underset{w_{M,t},h_{M,t},}{Max}\ (S_t^W)^d (S_t^F)^{1-d}$$

are:

$$(1-d)\frac{\partial\left(S_t^F\right)}{w_{M,t}}\left(S_t^W\right) + d\frac{\partial\left(S_t^W\right)}{\partial w_{M,t}}\left(S_t^J\right) = 0 \tag{A.5}$$

$$(1-d)\frac{\partial\left(S_t^F\right)}{\partial h_{M,t}}\left(S_t^W\right) + d\frac{\partial\left(S_t^W\right)}{\partial h_{M,t}}\left(S_t^J\right) = 0 \tag{A.6}$$

From these equation, and noting that $\frac{\partial^2 Y_t}{\partial N_t \partial h_{M,t}} = MP_{M,t}$, the allocation rule for regular hours obtains: $MP_{M,t} = \frac{(1+\tau_{N,t})}{(1-\tau_Y)} MRS_{M,t}$. As for the regular wage, by making use of equation (A.5), the free entry condition ($S_t^V = 0$) and the surpluses equations:

$$\begin{aligned} S_t^F &= \frac{\partial y_t}{\partial n_t} - (1+\tau_{N,t}) w_{M,t} h_{M,t} - (1 + p_D s_D \tau_{N,t}) w_{U,t} h_{U,t} + (1-\delta)\frac{\kappa}{q_t} \\ S_t^W &= (1-\tau_Y)\, w_{M,t} h_{M,t} + w_U h_{U,t} - \frac{B_0 (h_{M,t} + h_{U,t})^{1+\psi}}{(1+\psi)\,\mu_t} - \frac{B_1 (h_{U,t})^{1+\psi}}{(1+\psi)\,\mu_t} \\ &\quad - b + \frac{d}{1-d}(1-\delta-\theta_t q_t)\frac{\kappa}{q_t} E_t \frac{(1-\tau_Y)}{(1+\tau_{N,t+1})} \end{aligned}$$

we obtain the wage equation (9).

TECHNICAL APPENDIX to:

Tackling undeclared work.
Suggestions from a business cycle model with search frictions
(Not for publication)

1. Households

The representative household solves the problem:

$$\max_{\{C_t;K_{t+1}\}_0^\infty} E_0 \sum_{t=0}^{\infty} \beta^t \mathcal{U}_t \tag{1}$$

$$\text{s.t.} \quad : \quad C_t + K_{t+1} = (1-\tau_Y)\left(w_{M,t} N_t h_{M,t} + (r_t + \delta_K) K_t\right) + (1 - N_t)\, b \tag{2}$$
$$+ w_U N_t h_{U,t} + \Pi^e + (1-\delta_K) K_t$$

where: $\mathcal{U}_t = \log(C_t) - \mathcal{V}_t$ and: $\mathcal{V}_t = B_0 \frac{(h_{M,t}+h_{U,t})^{1+\psi}}{1+\psi} + B_1 \frac{(h_{U,t})^{1+\psi}}{1+\psi}$.

The first order conditions for a maximum of problem (1)-(2) are:

$$\left\{ \begin{array}{c} (C_t)^{-1} = \mu_t \\ \beta E_t \left\{ \mu_{t+1} \left[(1-\delta_K) + (1-\tau_Y)(r_{t+1} + \delta_K) \right] \right\} = \mu_t \\ \lim_{T\to\infty} E_0 \mu_T K_T = 0. \end{array} \right\}$$

where μ_t is Lagrange multiplier. The marginal rates of substitutions between consumption, on the one side, and regular or undeclared hours worked, on the other side, are respectively:

$$\begin{aligned} MRS_{M,t} &= \frac{\partial \mathcal{U}_t / \partial h_{M,t}}{\partial \mathcal{U}_t / \partial C_t} = C_t B_0 \left(h_{M,t} + h_{U,t}\right)^{\psi} \qquad (3) \\ MRS_{U,t} &= \frac{\partial \mathcal{U}_t / \partial h_{U,t}}{\partial \mathcal{U}_t / \partial C_t} = C_t \left[B_0 (h_{M,t} + h_{U,t})^{\psi} + B_1 (h_{U,t})^{\psi} \right] \end{aligned}$$

2. CES marginal productivities

The production function is:

$$Y_t = (K_t)^{\alpha} \left[(1-\omega)(\vartheta_t \cdot N_t h_{M,t})^{\rho} + \omega (N_t h_{U,t})^{\rho} \right]^{\frac{1-\alpha}{\rho}} \tag{4}$$

From straightforward computation relative to the production function (4), we obtain the following definitions:

$$\begin{aligned} \frac{\partial Y_t}{\partial (N_t)} &= (1-\alpha) \frac{Y_t}{N_t} = MP_{(N)} \qquad (5) \\ \frac{\partial^2 Y_t}{\partial (N_t) \partial h_{X,t}} &= MP_{X,t}; \text{ for: } X = M, U \\ MP_{M,t} &= \vartheta_t (1-\alpha)^2 \frac{Y_t}{\vartheta_t N_t h_{M,t}} \left(\frac{(1-\omega)(\vartheta_t N_t h_{M,t})^{\rho}}{(1-\omega)(\vartheta_t N_t h_{M,t})^{\rho} + \omega (N_t h_{U,t})^{\rho}} \right) \\ MP_{U,t} &= (1-\alpha)^2 \frac{Y_t}{N_t h_{U,t}} \left(\frac{\omega (N_t h_{U,t})^{\rho}}{(1-\omega)(\vartheta_t N_t h_{M,t})^{\rho} + \omega (N_t h_{U,t})^{\rho}} \right) \end{aligned}$$

3. The Job creating condition and the demand for capital

The expected profit of the firm is:

$$\Pi_t^e = Y_t - (1+\tau_{N,t}) w_{M,t} N_t h_{M,t} - (1+p_D s_D \tau_{N,t}) w_{U,t} N_t h_{U,t} - (r_t + \delta_K) K_t - \kappa V_t \tag{6}$$

The representative firm solves the following problem:

$$F_t^V(N_t) = \max_{V_t, K_t, N_t} \left[\Pi_t^e + \beta E_t \frac{\mu_{t+1}}{\mu_t} F_{t+1}^V(N_{t+1}) \right] \tag{7}$$

$$s.t.\ (4),\ (6),\ N_t = (1-\delta) N_{t-1} + q_{t-1} V_{t-1}$$

The first order condition with respect to V_t is:

$$\frac{\kappa}{q_t} = \beta E_t \frac{\mu_{t+1}}{\mu_t} \frac{\partial F_{t+1}^V(.)}{\partial N_{t+1}} \tag{8}$$

where we have used $\frac{\partial N_{t+1}}{\partial V_t} = q_t$. From the envelope theorem, we get:

$$\frac{\partial F_t^V(.)}{\partial N_t} = \frac{\partial Y_t}{\partial N_t} - (1+\tau_{N,t}) w_{M,t} h_{M,t} - (1+p_D s_D \tau_{N,t}) w_{U,t} h_{U,t} + \beta(1-\delta) E_t \frac{\mu_{t+1}}{\mu_t} \frac{\partial F_{t+1}^V(.)}{\partial N_{t+1}} \tag{9}$$

where we made use of $\frac{\partial N_{t+1}}{\partial N_t} = (1-\delta)$. Substitute equation (8) into (9) and update the resulting equation one period ahead to obtain:

$$\frac{\partial F_{t+1}^V(.)}{\partial N_{t+1}} = \frac{\partial Y_{t+1}}{\partial N_{t+1}} - (1+\tau_{N.t+1}) w_{M,t} h_{M,t} - (1+p_D s_D \tau_{N,t+1}) w_{U,t+1} h_{U,t+1} + (1-\delta) \frac{\kappa}{q_{t+1}}$$

Use this expression into equation (8) to get the job creating condition:

$$\frac{\kappa}{q_t} = \beta E_t \frac{\mu_{t+1}}{\mu_t} \left[\begin{array}{c} \frac{\partial Y_{t+1}}{\partial N_{t+1}} - (1+\tau_{N,t+1}) w_{M,t+1} h_{M,t+1} \\ -(1+p_D s_D \tau_{N,t+1}) w_{U,t+1} h_{U,t+1} + (1-\delta) \frac{\kappa}{q_{t+1}} \end{array} \right] \tag{10}$$

Finally, from the first order condition of (7) with respect to K_t, firms' demand of capital is given by:

$$\frac{\partial Y_t}{\partial K_t} = \alpha \frac{Y_t}{K_t} = r_t + \delta_K$$

4. Undeclared wages and hours

As for the determination of underground activities, the firm chooses the desired level of $h_{U,t}$ by maximizing the value of the marginal worker employed in production:

$$\max_{h_{U,t}} \frac{\partial Y_t}{\partial N_t} - (1+\tau_N) w_{M,t} h_{M,t} - (1+p_D s_D \tau_N) w_{U,t} h_{U,t} + (1-\delta) \frac{\kappa}{q_t}$$

where $w_{U,t}$, $w_{M,t}$ and $h_{M,t}$ are taken as given. The optimality condition is:

$$w_{U,t} = \frac{1}{(1 + p_D s_D \tau_N)} MP_{U,t} \tag{11}$$

where: $\frac{\partial^2 Y_t}{\partial (N_t) \partial h_{X,t}} = MP_U$.

The household chooses the desired level of $h_{U,t}$ (for given $w_{U,t}$, $w_{M,t}$ and $h_{M,t}$) by maximizing (1) subject to the constraint (2). This gives the optimality condition:

$$w_{U,t} = MRS_{U,t} \tag{12}$$

By equating the two optimality conditions, we obtain the (privately efficient) allocation rule for underground hours worked:

$$MRS_{U,t} = \frac{1}{(1 + p_D s_D \tau_N)} MP_{U,t} \tag{13}$$

5. Wages and Hours determination: declared work

Denote with $S_t^F = S_t^J - S_t^V$ the firms' surplus from an existing employment relationship where S_t^J and S_t^V are the value for a firm of a filled and an open vacancy, respectively, and are given by:

$$\begin{aligned} S_t^J \quad = \quad & \frac{\partial Y_t}{\partial N_t} - (1 + \tau_{N,t}) w_{M,t} h_{M,t} - (1 + p_D s_D \tau_{N,t}) w_{U,t} h_{U,t} \\ & + \beta E_t \frac{\mu_{t+1}}{\mu_t} \left[\delta S_{t+1}^V + (1 - \delta) S_{t+1}^J \right] \end{aligned} \tag{14}$$

and

$$S_t^V = -\kappa + \beta E_t \frac{\mu_{t+1}}{\mu_t} \left[q_t S_{t+1}^J + (1 - q_t) S_{t+1}^V \right] \tag{15}$$

Denote with $S_t^W = S_t^M - S_t^N$ the workers' surplus from an existing match, where S_t^M and S_t^N, are the values the worker enjoys when being matched, and unemployed, respectively:

$$\begin{aligned} S_t^M \quad = \quad & (1 - \tau_Y) w_{M,t} h_{M,t} + w_{U,t} h_{U,t} - \frac{B_1 (h_{U,t})^{1+\psi}}{(1+\psi) \mu_t} \\ & - \frac{B_0 (h_{M,t} + h_{U,t})^{1+\psi}}{(1+\psi) \mu_t} + \beta E_t \frac{\mu_{t+1}}{\mu_t} \left\{ (1 - \delta) S_{t+1}^M + \delta S_{t+1}^N \right\} \end{aligned} \tag{16}$$

and

$$S_t^N = b + \beta E_t \frac{\mu_{t+1}}{\mu_t} \left[\theta_t q_t S_{t+1}^M + (1 - \theta_t q_t) S_{t+1}^N \right] \tag{17}$$

Firms and workers negotiate on the regular input so as to maximize their weighted joint surplus:

$$\underset{w_{M,t}, h_{M,t},}{Max} (S_t^W)^d (S_t^F)^{1-d}$$

where $w_{U,t}$ and $h_{U,t}$ are taken as given. Considering that the free entry condition implies $S_t^V = 0$, the corresponding first order conditions for $w_{M,t}$ and $h_{M,t}$, are, respectively:

$$w_{M,t}: \quad (1-d)\gamma_t^F \left(S_t^W\right) + d\gamma_t^W \left(S_t^J\right) = 0 \tag{18}$$

$$h_{M,t}: \quad (1-d)\lambda_t^F \left(S_t^W\right) + d\lambda_t^W \left(S_t^J\right) = 0 \tag{19}$$

where γ_t^i, λ_t^i $i \in \{F, W\}$ represent the marginal effects of the bargained variables over the firms' and households' surplus, respectively, and are given by:

$$\gamma_t^F = \frac{\partial\left(S_t^J\right)}{w_{M,t}} = -(1+\tau_{N,t})h_{M,t} \tag{20}$$

$$\gamma_t^W = \frac{\partial\left(S_t^M - S_t^N\right)}{\partial w_{M,t}} = (1-\tau_Y)\, h_{M,t} \tag{21}$$

$$\lambda_t^F = \frac{\partial\left(S_t^J\right)}{\partial h_{M,t}} = \frac{\partial^2 Y_t}{\partial N_t \partial h_{M,t}} - (1+\tau_{N,t})w_{M,t} \tag{22}$$

$$\lambda_t^W = \frac{\partial\left(S_t^M - S_t^N\right)}{\partial h_{M,t}} = (1-\tau_Y)\, w_{M,t} - MRS_{M,t} \tag{23}$$

In order to solve the system of the optimality conditions we need to simplify the equations of the agents' surplus. By imposing the free entry condition ($S_t^V = 0$) in equation (15) and taking into account the definition of (14), the firm's surplus can be written as:

$$S_t^F = S_t^J = \frac{\partial Y_t}{\partial N_t} - (1+\tau_{N,t})w_{M,t}h_{M,t} - (1+p_D s_D \tau_{N,t})w_{U,t}h_{U,t} + (1-\delta)\frac{\kappa}{q_t} \tag{24}$$

Using equations (16) and (17), the workers surplus from an existing match can be written as follows:

$$\begin{aligned} S_t^W &= \left(S_t^M - S_t^N\right) = (1-\tau_Y)\, w_{M,t}h_{M,t} + w_U h_{U,t} - \frac{B_1(h_{U,t})^{1+\psi}}{(1+\psi)\,\mu_t} \\ &\quad - \frac{B_0(h_{M,t}+h_{U,t})^{1+\psi}}{(1+\psi)\,\mu_t} - b + \beta(1-\delta-\theta_t q_t)E_t\frac{\mu_{t+1}}{\mu_t}S_{t+1}^W \end{aligned} \tag{25}$$

From equations (8) and (18), the future expected net present value from employment can be written as follows:

$$S_{t+1}^W = \frac{d}{1-d}\frac{(1-\tau_Y)}{(1+\tau_{N,t+1})}S_{t+1}^J;$$

which we substitute into (25) to obtain:

$$\begin{aligned} S_t^W &= (1-\tau_Y)\, w_{M,t}h_{M,t} + w_U h_{U,t} - \frac{B_0(h_{M,t}+h_{U,t})^{1+\psi}}{(1+\psi)\,\mu_t} - \frac{B_1(h_{U,t})^{1+\psi}}{(1+\psi)\,\mu_t} - b \\ &\quad + (1-\delta-\theta_t q_t)\frac{d}{1-d}E_t\beta\frac{\mu_{t+1}}{\mu_t}S_{t+1}^J\frac{(1-\tau_Y)}{(1+\tau_{N,t+1})} \end{aligned}$$

Finally, using (8) we get:

$$S_t^W = (1-\tau_Y)\, w_{M,t} h_{M,t} + w_U h_{U,t} - \frac{B_1 (h_{U,t})^{1+\psi}}{(1+\psi)\,\mu_t} - b \tag{26}$$
$$-\frac{B_0 (h_{M,t}+h_{U,t})^{1+\psi}}{(1+\psi)\,\mu_t} + \frac{d}{1-d}(1-\delta-\theta_t q_t)\frac{\kappa}{q_t} E_t \frac{(1-\tau_Y)}{(1+\tau_{N,t+1})}$$

Now consider again:

$$S_t^W = \frac{d}{1-d}\frac{(1-\tau_Y)}{(1+\tau_{N,t})} S_t^J \tag{27}$$

by substituting (24) and (26) into (27) we get the following equation for the regular wage:

$$w_{M,t} = \frac{(1-d)}{(1-\tau_Y)\, h_{M,t}}\left(\frac{\mathcal{V}_t}{\mu_t} + b - w_U h_{U,t}\right) \tag{28}$$
$$+\frac{d}{(1+\tau_{N,t}) h_{M,t}}\left(\frac{\partial Y_t}{\partial N_t} - (1+p_D s_D \tau_{N,t}) w_{U,t} h_{U,t}\right)$$
$$+\frac{d}{(1+\tau_{N,t}) h_{M,t}}\left[1-\left(1-\frac{\theta_t q_t}{(1-\delta)}\right)\frac{(1+\tau_{N,t})}{(1+\tau_{N,t+1})}\right](1-\delta)\frac{\kappa}{q_t}$$

As for bargained hours, we now substitute (27) into equation (19) to get:

$$(1-d)\lambda_t^F \frac{d}{1-d}\frac{(1-\tau_Y)}{(1+\tau_{N,t})} S_t^J + d\lambda_t^W S_t^J = 0$$

By using (22) and (23), and recalling that $\frac{\partial^2 Y_t}{\partial N_t \partial h_{M,t}} = MP_{M,t}$, the above equation can be written as:

$$MP_{M,t} = \frac{(1+\tau_{N,t})}{(1-\tau_Y)} MRS_{M,t} \tag{29}$$

By dividing (13) by (29), and making use of the expressions in (5) and (3), the following equation:

$$\frac{MP_{U,t}}{MP_{M,t}} = \frac{(1+p_D s_D \tau_{N,t})\,(1-\tau_Y)}{(1+\tau_{N,t})}\frac{MRS_{U,t}}{MRS_{M,t}}$$

gives raise to:

$$(h_{R,t})^{\rho-1} = \frac{\vartheta_t^\rho\,(1-\omega)\,(1-\tau_Y)\,(1+p_D s_D \tau_{N,t})}{\omega(1+\tau_{N,t})}\left[1+\frac{B_1}{B_0}\left(1+(h_{R,t})^{-1}\right)^{-\psi}\right] \tag{30}$$

7. Stationary state equations.

The model's stationary state equations are the following ones.

Labor market:

$$M^* = \delta N^*$$
$$p^* = M^*/U^*$$
$$M^* = \eta(V^*)^{\xi}(U^*)^{1-\xi}$$
$$\theta^* = V^*/u^*$$
$$q^* = \frac{M^*}{V^*}$$
$$U^* = 1 - N^*$$

Households:

$$\frac{1}{C^*} = \mu^*$$
$$r^* = \left[\frac{1}{\beta} - 1 + \delta_k\right] \frac{1}{(1-\tau_Y)} - \delta_K > 0$$
$$MRS^*_{_M} = C^* B_0 \left(h^*_M + h^*_U\right)^{\psi}$$
$$MRS^*_{_U} = C^* \left[B_0(h^*_M + h^*_U)^{\psi} + B_1(h^*_U)^{\psi}\right]$$

Firms:

$$Y^* = (K^*)^{\alpha} \left(N^*\right)^{1-\alpha} \left[(1-\omega)\left(\vartheta h^*_M\right)^{\rho} + \omega \left(h^*_U\right)^{\rho}\right]^{\frac{1-\alpha}{\rho}}$$
$$\frac{\left[1-\beta\left(1-\delta\right)\right]\kappa}{\beta q^*} = (1-\alpha)\frac{Y^*}{N^*} - (1+\tau^*_N) w^*_M h^*_M - (1 + p_D s_D \tau^*_N) w^*_U h^*_U$$
$$r^* = \alpha\left(\frac{Y}{K}\right)^* - \delta_K$$
$$MP^*_M = (1-\alpha)^2 \frac{Y^*}{h^*_M N^*} \frac{(1-\omega)\left(h^*_M \vartheta^*\right)^{\rho}}{(1-\omega)\left(h^*_M\vartheta^*\right)^{\rho} + \omega\left(h^*_U\right)^{\rho}}$$
$$MP^*_U = (1-\alpha)^2 \frac{Y^*}{h^*_U N^*} \frac{\omega\left(h^*_U\right)^{\rho}}{(1-\omega)\left(h^*_M\vartheta^*\right)^{\rho} + \omega\left(h^*_U\right)^{\rho}}$$

Hours and wages

$$
\begin{aligned}
MRS_U^* &= \frac{1-\alpha}{1+p_D s_D \tau_N^*} MP_U^* \\
MRS_M^* &= \frac{(1-\alpha)(1-\tau_Y)}{1+\tau_N^*} MP_M^* \\
w_U^* &= \frac{1}{1+p_D s_D \tau_N^*} MP_U^* \\
w_M^* &= \frac{(1-d)}{(1-\tau_Y)\, h_M^*} Q_1^* + \frac{d}{(1+\tau_N^*) h_M^*} Q_2^* \\
Q_1^* &= \frac{\mathcal{V}^*}{\mu^*} + b - w_U^* h_U^* \\
Q_2^* &= \frac{\partial Y^*}{\partial N^*} - (1+p_D s_D \tau_N^*) w_U^* h_U^* + \kappa \theta^* \\
\mathcal{V}^* &= B_0 \frac{(h_M^* + h_U^*)^{1+\psi}}{1+\psi} + B_1 \frac{(h_U^*)^{1+\psi}}{1+\psi}.
\end{aligned}
$$

Constraints

$$C^* + \delta_K K^* = Y^* - G^* - \kappa V^*$$

$$G^* = \tau_Y \left[w_M^* N^* h_M^* + (r^* + \delta_K) K^* \right] + s_D p_D \tau_N^* w_U^* N^* h_U^* + \tau_N^* N^* h_M^* w_M^* - b\,(1-N^*)$$

The system given by these 24 equations is solved numerically.

The steady state underground share S_U^* is defined as $S_U^* = \frac{w_U^* h_U^* N^*}{Y^*}$. Use equations (11) together with the definitions in (5) to obtain:

$$w_U^* = \frac{(1-\alpha)^2}{(1+p_D s_D \tau_N^*)} \frac{Y^*}{N^* h_U^*} \frac{\omega\,(h_U^*)^\rho}{(1-\omega)\,(\vartheta^* h_M^*)^\rho + \omega\,(h_U^*)^\rho}$$

Substitute this value of w_U^* into S_U^* and obtain:

$$S_U^* = \frac{w_U^* h_U^* N^*}{Y^*} = \frac{(1-\alpha)^2}{(1+p_D s_D \tau_N^*)} \left(\frac{\omega\,(h_R^*)^\rho}{(1-\omega)\,(\vartheta^*)^\rho + \omega\,(h_R^*)^\rho} \right)$$