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How responsive are real exchange rates in developing countries to terms of trade shocks?

Lavan Mahadeva[1,*], Juan Carlos Parra Alvarez[2]

Abstract

In developing countries, policy assessments about the reaction of the domestic economy to external shocks depend on the degree of substitution between domestic and imported items. We estimate these key parameters for Colombia. We find that the input of the distribution sector in transforming imports is complementary, implying a great potential sensitivity of the Colombian economy to external shocks. However we also find that consumption imports at the point of sale are substitutes with domestic items. Even though the distribution sector is smaller, its influence may dominate because the responsiveness of the real exchange rate is highly nonlinear in the elasticity.

Keywords: Exchange Rate Policy, Elasticity, Colombia, Kalman Filter.
JEL Classification: F47, E01, C61.

1. Introduction

In small open economies, the policy assessment of the impact external shocks depend crucially on the degree of substitution between domestic and imported goods. We follow Devaragan et al. (1990) in demonstrating that the response of the real exchange rate to a terms of trade shock is sensitive to how complementary imports are to domestically produced items in consump-

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tion. But McDaniel & Balistreri (2003) also document that many of the answers that general equilibrium models traditionally provide to trade policy questions hinge on these elasticities, traditionally called Armington elasticities after Armington (1969)'s seminal paper.

As developing countries have continued to open up to trade in goods and capital, there is an ever greater interest on policy questions which depend on these elasticities. For instance Abrego & Whalley (2000) demonstrate that these critical elasticities shape how trade openness affects wage inequality. Yet another illustration concerns import price passthrough. As the domestic output gap influences the final price of imported goods through imported domestically produced margins of commerce on imported goods (Côté et al., 2006), the more complementary the role of the distribution sector in the transport, marketing and sale of imported goods, the greater the leverage of domestic conditions to dampen any persistent imported price rises. As a final example, a recent paper shows that the elasticity of demand for nontradables and tradables could critically determine the scale of the capital outflow and real exchange rate depreciation during a financial crisis in a developing country with accumulated debt (Bianchi, 2009, pp.23).

Naturally then the models used to provide policy advice for small open economies split production into aggregated tradable and nontradable sectors and put these elasticities on centre stage. See for example, the open economy models described in Obstfeld & Rogoff (1996) or Galí & Monacelli (2005)'s Dynamic Stochastic General Equilibrium model for open economies.

This paper estimates the substitutability between domestically produced and imported goods in order to better assess the policy options of developing countries in the face of external shocks. Taking Colombia to be a case study, we find that the input of the distribution sector is complementary in transforming imports for final consumption. The value would suggest that the real exchange rate can appreciate strongly in response to a positive terms of trade shock. Imported investment goods are also complementary in combining with domestic investment goods. But imported consumer goods are substitutes with domestic consumer goods and imported raw materials also combine with consumer and investment imports with an elasticity not far above the Cobb Douglas values. Both of these findings indicate that the real exchange rate would respond little or may even depreciate. We show that the responsiveness of the real exchange rate to a terms of trade shock is nonlinear, and so it may not be that the high consumption elasticity offsets the low distribution sector elasticity. Our estimates allow for measurement error in creating sectoral data as well as the possibility of a variety of trends that we find in the the data, such as population growth, aggregate and sector-specific technological progress trends, and changes in preferences or technology. Crucially, we also test if those estimates are robust. Robustness is especially important because these parameters should be estimated at least at the level of disaggregate sectors to avoid bias caused by aggregation which typically has the effect of pushing estimates towards zero (Imbs & Mejean (2009)). But data on the disaggregate sectors have to be purpose-built: National Accounts offices do not publish data on the real volumes and deflators of these aggregate sectors. We find that the estimates of consumption elasticities are very robust and of the distribution sector quite robust. The estimates of the imported investment sensitivity are likely to be unreliable.

The paper is organized as follows. The theoretical foundation of these equations is explained in the next section, Section 2. In Section 3 we describe how we adapt Colombian National Accounts data to prepare a database for this model. Section 4 motivates our signal in broad terms and then justifies its particular design. Results are reported for five individual demand relationships in Section 5. Section 6 concludes.

2. Theory

2.1. Stylised illustration

A stylised model, adapted from Devaragan et al. (1990), can bring out the importance of these elasticities for policy preoccupations. Let the demand for imported goods (M_t) as opposed to domestically produced items (D_t) be described by the general function

$$\frac{M_t}{D_t} = f\left(\frac{P_{mt}}{P_{dt}}\right) \tag{1}$$

where P_{mt} and P_{dt} are the price of imports and domestically produced goods respectively. Similarly the supply for export (E_t) as opposed to that destined for domestic consumption is given by

$$\frac{E_t}{D_t} = g\left(\frac{P_{et}}{P_{dt}}\right) \tag{2}$$

where P_{et} is the domestic price of exports. The trade balance as a share of the value of GDP ($\equiv P_{dt}D_t + P_{et}E_t$) is defined as

$$\frac{B_t}{P_{dt}D_t + P_{et}E_t} = \left(\frac{P_{et}}{P_{dt}}\frac{E_t}{D_t} - \frac{P_{mt}}{P_{dt}}\frac{M_t}{D_t}\right)\frac{P_{dt}D_t}{P_{dt}D_t + P_{et}E_t}. \tag{3}$$

The aim is to describe the effect of a terms of trade shock on the real exchange rate, assuming that the trade balance as a share of GDP is constant. Substituting into equation 3 from equations 1 and 2 for M_t, D_t and E_t, and differentiating and rearranging, yields

$$\frac{drer_t}{dtot_t}\frac{tot_t}{rer_t} = \frac{1-\omega}{\left((1-\omega) - \frac{DD_t}{GDP_t}\frac{P_{et}E_t}{P_{mt}M_t}(1-\phi)\right)} \tag{4}$$

where the terms of trade is defined as $tot_t \equiv \frac{P_{et}}{P_{mt}}$, the real exchange rate is $rer_t \equiv \frac{P_{et}}{P_{dt}}$, $DD_t \equiv P_{dt}D_t + P_{mt}M_t$ is domestic demand, ϕ is the elasticity of supply (equation 2)and ω is the focus of this paper, the elasticity of demand between domestically produced and imported goods from equation 3. If $\omega = 0$, the two items are Leontieff complements, if $\omega = 1$, they are Cobb-Douglas complements, and if $\omega = \infty$, they are perfect substitutes.

In general, the elasticity of the real exchange rate to the terms of trade depends on the the degree of substitution between domestic and imported goods with the trade balance held to a fixed share of GDP. If the elasticity is unitary, the Cobb Douglas value, there will be no response. Otherwise the response can vary in direction and scale. If exports and domestic items are complements in production, as is likely, the relationship is monotonically increasing, such that the more complementary are imported goods in demand, the greater the appreciation we are likely to see from a positive terms of trade shock. But the relationship is also highly nonlinear. Consider what would happen if it falls within the range:

$$1 - \frac{DD_t}{GDP_t}\frac{P_{et}E_t}{P_{mt}M_t}(1-\phi) < \omega < 1. \tag{5}$$

5 is certainly possible. That ω lies above the lower bound is plausible because domestic demand is not likely to be much greater than GDP, and exports not much greater than imports, while the elasticity of export supply is likely to be smaller than the elasticity of demand for imports relative to domestic goods. And as we shall see , we can find values of ω below the Cobb-Douglas value of 1, keeping below the upper bound.

The point is that if ω lies in the range 5, a positive terms of trade shock will imply a real appreciation ($\frac{drer_t}{dtot_t}\frac{tot_t}{rer_t} < 0$), and with the denominator potentially small, that appreciation can be large. The intuition is that when the terms of trade improve, there must be a rise in the share of imports in domestic consumption to match the rise in the value of exports in production so as to keep the trade balance fixed. If imports are complementary in consumption, but export price elasticities are even lower, then the price of domestic items must rise to be consistent with the greater share of spending on imports. That rise, above the domestic currency export price, implies a real appreciation.

In summary, this simple model illustrates that policy assessment of international shocks in open economies has to be based on an assessment of the elasticity of demand for domestic goods. This would account for the fact that the production of goods and services sold for final or intermediate consumption is vertically disintegrated across the national border and where the distribution sector plays a potentially important role.

In the rest of this section, the key relationships that can be used to estimate these elasticities are derived from theory. Although these relations can be couched in a general equilibrium model,we do not do that here. In what follows, the convention is to denote per capita volumes by lower case and aggregate volumes by upper case. Our specification and estimation strategy is together general enough to allow for realistic features such as population growth and technical progress and relative price trends. It is also worth noting that the relationship between the real exchange rate terms of trade response and the elasticity of substitution of imports is highly nonlinear, and therefore it may not be the case that the effect of an elasticity just below the Cobb Douglas value in one sector is offset by a value of just above zero in another even more important sector. As we shall see this might very well be the case with Colombia.

2.2. Distribution

We first describe the role of the transport and distribution sector (henceforth just the distribution sector) in taking imports of consumer and investment goods from outside the national frontier, transporting them, storing them, marketing them and selling them to the consumer. Imports at the dock are combined with the output of the domestic distribution with both then taken as intermediate inputs that deliver the final consumption item. The price of the domestic input is the margin. Burstein et al. (2003) found that in the case of Argentina, distribution margins are more than 40% of the retail price of the average consumer good. They argue that allowing for this, as one should, is critical to understanding the behaviour of the real exchange rate during exchange-rate-based stabilizations.

In what follows T_t is the real domestic input of the distribution sector, M_t^d is the volume of imported consumption and investment goods at the point of use, and M_t^p is the volume of these goods at the docks, before transformation. P_t^T, P_t^{md} and P_t^{mp} are the respective prices. The transformation function is:

$$M_t^d = z_t^m M\left(T_t, M_t^p\right) = z_t^m \left[\iota_{Tt}^{\frac{1}{a_m}} \left(T_t\right)^{\frac{a_m-1}{a_m}} + \left(1-\iota_{Tt}\right)^{\frac{1}{a_m}} \left(M_t^p\right)^{\frac{a_m-1}{a_m}}\right]^{\frac{a_m}{a_m-1}} \tag{6}$$

with $a_m \geq 0$ is the elasticity of substitution between the domestically produced distribution input and foreign imports, taking exactly the same range of values as in equation 4.

The maximisation problem of these firms is

$$\begin{aligned} \max_{\{T_t,\, M_t^p\}} \quad & P_t^{md} M_t^d - P_t^T T_t - P_t^{mp} M_t^p \\ \text{s.t.} \quad & M_t^d \leq z_t^m \left[\iota_{Tt}^{\frac{1}{a_m}} \left(T_t\right)^{\frac{a_m-1}{a_m}} + \left(1-\iota_{Tt}\right)^{\frac{1}{a_m}} \left(M_t^p\right)^{\frac{a_m-1}{a_m}}\right]^{\frac{a_m}{a_m-1}} \end{aligned} \tag{7}$$

The solution to this problem yields the following relationship between the share parameters in the Constant Elasticity of Substitution (CES) consumption function and the relative price and nominal share of spending on domestically produced items, which will form the basis of our estimate of the key parameter a_m:

$$\frac{P_t^T T_t}{P_t^{md} M_t^d} = (z_t^m)^{a_m - 1} \left(\frac{P_t^T}{P_t^{md}} \right)^{1-a_m} \iota_{Tt}. \tag{8}$$

2.3. Consumption

In a similar fashion, a formal description of the representative consumer's decision can yield equations to estimate the elasticity of substitution between domestically produced and foreign produced consumer goods. Total consumption can be described as a CES aggregate of the consumption of domestically produced $\left(c_t^d\right)$ and foreign produced items (c_t^m),

$$c_t = c\left(c_t^d, c_t^m\right) = \left[\gamma_t^{\frac{1}{\omega}} \left(c_t^d\right)^{\frac{\omega-1}{\omega}} + (1-\gamma_t)^{\frac{1}{\omega}} \left(c_t^m\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}. \tag{9}$$

The solution to the problem of allocating consumption between foreign and domestically produced items subject to a budget constraint is then expressed in share and relative price terms:

$$\frac{P_t^{cd} C_{Dt}}{P_t^c C_t} = \left(\frac{P_t^{cd}}{P_t^c} \right)^{1-\omega} \gamma_t, \tag{10}$$

with P_t^{cd} and P_t^c being the respective prices of domestic items and of total consumption.

2.4. Investment

The substitutability of domestically produced investment goods, such as structures, with foreign produced investment goods, such as machinery, also matters in determining the responsiveness of the economy to external shocks.

As an approximation, we assume that domestic producers create an aggregate investment good by combining domestically produced investment, X_t^d, with foreign produced investment, X_t^m,:

$$X_t = z_t^x X\left(X_t^d, X_t^m\right) = z_t^x \left[(\varkappa_t)^{\frac{1}{\iota}} \left(X_t^d\right)^{\frac{\iota-1}{\iota}} + (1-\varkappa_t)^{\frac{1}{\iota}} \left(X_t^m\right)^{\frac{\iota-1}{\iota}} \right]^{\frac{\iota}{\iota-1}} \tag{11}$$

where X_t is aggregate gross investment. P_t^{xd}, P_t^{md} and P_t^x are the deflators of domestic, imported and total investment goods respectively. The maximisation problem is:

$$\begin{array}{ll} \max & P_t^x X_t - P_t^{xd} X_t^d - P_t^m X_t^m \\ \text{s.t.} & x_t \leq z_t^x \left[(\varkappa_t)^{\frac{1}{\iota}} \left(X_t^d\right)^{\frac{\iota-1}{\iota}} + (1-\varkappa_t)^{\frac{1}{\iota}} \left(X_t^m\right)^{\frac{\iota-1}{\iota}} \right]^{\frac{\iota}{\iota-1}}. \end{array} \tag{12}$$

The solution implies the familiar share and relative price equation:

$$\frac{P_t^{xd} X_t^d}{P_t^x X_t} = \left(z_t^x\right)^{\iota-1} \left(\frac{P_t^{xd}}{P_t^x}\right)^{1-\iota} \varkappa_t. \tag{13}$$

2.5. Raw material imports

It seems plausible that raw material imports require less transformation by the distribution sector before consumption than imported capital and consumption goods. To test then if imported raw materials are a complementary input into the economy, we estimate the elasticity of raw materials relative to imports of consumer and investment goods in being disaggregated from total imports.

P_t^{um} is the total imports deflator from national accounts, that is, the price of imports before transformation by the distribution sector. M_t^u is the equivalent volume of imports at the dock. The price of consumption and capital imports before transformation is P_t^{mp}, the domestic currency raw material price before transformation is P_t^{rm} and the volume of raw materials is RM_t.

The maximisation problem is then:

$$\begin{array}{ll} \max\limits_{\{M_t^p,\, RM_t\}} & P_t^{um} M_t^u - P_t^{rm} RM_t - P_t^{mp} M_t^p \\ \text{s.t.} & M_t^u \leq z_t^{rm} \left[(\iota_{rmt})^{\frac{1}{ar}} \left(RM_t\right)^{\frac{a_r-1}{a_r}} + (1-\iota_{rmt})^{\frac{1}{ar}} \left(M_t^p\right)^{\frac{a_r-1}{a_r}} \right]^{\frac{a_m}{a_m-1}} \end{array} \tag{14}$$

The solution implies

$$\frac{P_t^{rm} RM_t}{P_t^{um} M_t^u} = \iota_{rmt} \left(z_t^{rm}\right)^{a_r - 1} \left(\frac{P_t^{rm}}{P_t^{um}}\right)^{1-a_r}. \tag{15}$$

2.6. The private sector consumer

Our model melds private sector consumers with the government. But we might want estimate a household-only consumption elasticity. Analogously to problem 9 and its solution 10, an equation for private sector's demand for domestically produced consumption is given as:

$$\frac{P_t^{cdp} C_{Dt}^p}{P_t^{cp} C_t^p} = \left(\frac{P_t^{cdp}}{P_t^{cp}}\right)^{1-\omega^p} \gamma_t^p, \tag{16}$$

with the price and volume of domestically produced consumption private sector written as P_t^{cdp} and C_{Dt}^{cp}, and the price and volume of total private consumption as P_t^{cp} and C_t^{cp}.

3. The data

Consider now the data we would need to estimate these crucial elasticities using equations 8, 10, 13, 15 and 16. We can divide our data needs into two. Table 1 includes the National Accounts expenditure aggregates, which are all available as published series as both nominal and real values for Colombia.

Table 1: National accounts data for the model

Variable	**Explanation**	**National Accounts available data**
P_t^c, C_t, $P_t^c C_t$	Consumption of households and government	Nominal, real volumes
P_t^x, X_t, $P_t^x X_t$	Gross fixed capital formation and changes in inventories	Nominal, real volumes
P_t^{um}, M_t^u, $P_t^{um} M_t^u$	Total imports before transformation	Nominal, real volumes
P_t^{cp}	Price of private sector consumption	CPI data
P_t^{rm}, RM_t, $P_t^{rm} RM_t$	Raw materials	Nominal, real volumes

A second table includes the variables for which there is no National Accounts counterpart.

Table 2: Missing data for the model

Variable	**Explanation**
$P_t^{cm}, C_t^m, P_t^{cm} C_t^m$	Consumption by households and government of direct imports
$P_t^{cd}, C_t^d, P_t^{cd} C_t^d$	Consumption by households and government of domestic production
$P_t^{cdp}, C_{Dt}^p, P_t^{cdp} C_{Dt}^p$	Consumption by households of domestic production
$P_t^{md}, M_t^d, P_t^{md} M_t^d$	Aggregate capital and consumption imports after transformation
$P_t^{mp}, M_t^p, P_t^{mp} M_t^p$	Aggregate capital and consumption imports before transformation
$P_t^T, T_t, P_t^T T_t$	Distribution sector input into transforming consumption and capital imports
$P_t^{xm}, X_t^m, P_t^{xm} X_t^m$	Imported physical investment
$P_t^{xd}, X_t^d, P_t^{xd} X_t^d$	Domestically produced physical investment

A popular method is to categorise sectoral output as either being a tradable or a nontradable sector. Most typically the goods producing sectors are classified as tradables and services as nontradables. Then the real volume and deflators series can be calculated as aggregates of these components. But we find this approach very problematic as many sectors contain both tradable and nontradable elements. On these grounds, we adopted a more complex tactic.

As a first step, we used the input-output tables and other national accounts estimates about the import intensity of each sector. Weighting by intensity and adding gave us annual nominal shares for some of the missing series in 2, such as imported consumption or the distribution sector input. We interpolated and extrapolated those shares to cover our whole sample at a quarterly frequency. Aside from this, we obtained data on either the price and volume split of other missing series from other parts of National Accounts data or from other sources. And we already had data on aggregates both as prices and volume: the series in 1.

To complete the exercise, we had to produce series on what was left over, either a missing component's price or volume. The reason for this is that even if we have data for the series P_{1t}, P_t, Z_t and $\frac{P_{1t} Z_{1t}}{P_t Z_t}$ in the formula

$$P_{1t} Z_{1t} + P_{2t} Z_{2t} = P_t Z_t,$$

that would not be enough to derive the component P_{2t} and Z_{2t} separately.

There are two ways to overcome this impasse. We could either make use of strong assumptions about relative prices, for example that they are fixed ($P_{1t} = P_{2t}$), or we could employ index number formulae. In the example above, an index formula could give Z_{2t} as a value-added component. Index number formulae are designed to be compatible with a wider range of utility and production functions than the specific CES forms assumed in our derivations (Diewert & Nakamura, 1993). And they do not require assumptions about particular values of elasticities, as would the CES aggregate implied by our theoretical derivations.

We used index numbers as much as possible because they suited our purpose well. But we still had to assume fixed relative prices between firms' inventories and their investment; between imported consumption and imported investment and in the government and private consumers' purchase of domestically produced goods. Bold assumptions such as these are unavoidable when building tradable nontradable sector models. At least our estimation strategy allowed us to highlight the risks to our estimates if these assumptions is not supported by the data.

4. Estimation strategy

In this section we explain how we estimated the crucial elasticities and tested their reliability. In a nutshell, each equation is estimated by regressing the nominal share of a nontradable sector in the total on its relative price. Our estimates of the key elasticities of substitution are based on the five pairs of equations that link relative price and shares across tradable and non-tradable sectors: the input of the distribution sector in transforming capital and consumer imports (equation 8); domestic consumption as a share of total consumption (equation 10); domestic investment relative to total investment (equation 13); raw materials relative to total imports (equation 15) and private sector domestic good consumption relative to total private consumption (equation 16).

In the case of the demand for consumption of domestic production versus imported consumption items, this equation can be written as:

$$s_{cdt} = \left(\frac{P_t^{cd}}{P_t^c}\right)^{1-\omega} \gamma_t \tag{17}$$

where s_{cdt} is the nominal share of domestically produced consumption in total consumption.

These demand and supply relations are a suitable basis from which assess our model-database combination. First as we are regressing a nominal share on a relative price, this relationship should work well even if the data features population changes, aggregate or sectoral productivity shifts. The relationship will only break down when there are problems in modelling these trends.

To see why, remember that in the case of domestic consumption, we did not use the theoretical price aggregator relationship to calculate the price index for domestically produced consumption but instead a more general index number formula. Thus if P_t^{cd} refers to the theoretically consistent domestic good consumption deflator and $P_t^{dat,cd}$ refers to our index, then the share of domestic consumption is given by:

$$s_{cdt} = \left(\frac{P_t^{cd}}{P_t^c}\right)^{1-\omega} \gamma_t$$

$$\Rightarrow s_{cdt} = \left(\frac{P_t^{dat,cd}}{P_t^c}\right)^{1-\omega} \gamma_t \left(\frac{P_t^{cd}}{P_t^{dat,cd}}\right)^{1-\omega}$$

$$\Rightarrow s_{cdt} = \left(\frac{P_t^{dat,cd}}{P_t^c}\right)^{1-\omega} \vartheta_t \tag{18}$$

and so a composite residual term comprises the measurement error and the true parameter, γ_t,

$$\vartheta_t \equiv \gamma_t \left(\frac{P_t^{cd}}{P_t^{dat,cd}}\right)^{1-\omega}.$$

If there were any mismatch between our relative price indices and the true data generating process, that would show up in the residual ϑ_t. This might happen if there were errors in our construction of nontradable and tradable price series. One reason could be an important intermediate trade between tradable and nontradable sectors which should in principle be modelled

(Valadkhani, 2004; Basu, 1995) but in practice is difficult to do. Another source of error could also be a shift in the technical progress in the production of domestically produced items relative to all other goods that is not captured in the data, especially where the quality of the goods are improved. For example in the equation for investment goods (13) there could be a shift in the term z_t^x.

ϑ_t may also include misspecifications in the model rather than the data. Another reason for a residual would be that the imposed model may excessively restrict preferences, such as imposing a constant γ_t when in reality this parameter shifts. For example the CES functional form may be at odds with a reality that requires more general formulae[3].

Our estimates are based on a state-space model of equation 18. We favour the state-space format because it can incorporate the desired features that ϑ_t can be a time-varying unobserved component and that the model includes some process for the relative price series.

We assume that ϑ_t follows the AR(1) state process:

$$\ln(\vartheta_t) = \phi_{11}\ln(\vartheta_{t-1}) + u_{1t}. \tag{19}$$

Then the observation equation of the state-space model is:

$$\mathbf{y}_t = \mathbf{H}\alpha_t \tag{20}$$

and the state equation is:

$$\alpha_t = \mathbf{\Phi}\alpha_{t-1} + \mathbf{\Xi} + \mathbf{u}_t \tag{21}$$

with

$$\alpha_t \equiv [\ln(\vartheta_t), \ln(x_t)]^T; \tag{22}$$

$$\mathbf{y}_t \equiv \begin{bmatrix} \ln(\frac{P_t^{cd}}{P_t^c}) & \ln(s_{cdt}) \end{bmatrix}^T; \tag{23}$$

[3]Fernández-Villaverde & Rubio-Ramírez (2007) present some other examples. Imbs & Mejean (2009) also estimate the same elasticities with very similar specifications, using similar arguments to ours.

$$\mathbf{u}_t \equiv \begin{bmatrix} u_{1t} & u_{2t} \end{bmatrix}^T; \tag{24}$$

$$\mathbf{\Phi} \equiv \begin{bmatrix} \phi_{11} & 0 \\ 0 & \phi_{22} \end{bmatrix}; \tag{25}$$

$$\mathbf{\Xi} \equiv \begin{bmatrix} (1-\phi_{11})\,\xi_1 \\ (1-\phi_{22})\,\xi_2 \end{bmatrix}; \tag{26}$$

$$\mathbf{Q} \equiv \begin{bmatrix} \sigma^2_{u1} & 0 \\ 0 & \sigma^2_{u2} \end{bmatrix}; \tag{27}$$

and

$$\mathbf{H} \equiv \begin{bmatrix} 0 & 1 \\ 1 & 1-\omega \end{bmatrix}. \tag{28}$$

The observation equations are first a simple definition which links the state process for the relative price to the data series and second, the share demand equation, equation 18, expressed in log terms. The two state equations describe how ϑ_t and the relative price, x_t, both in logs, evolve. Thus the model allows for a time-varying ϑ_t and a time-varying share, and for the two to be cointegrated jointly with the relative price. All unobserved stochastic variation in the relationship is subsumed in ϑ_t, including any measurement error.

But even a simple state-space model such as this can involve some severe identification problems. Using data on the relative price and the share only it is difficult to jointly identify all the three constants and three variances. To overcome this we adopted a two-step approach. We first estimate an AR(1) process for relative prices by ordinary least squares (OLS):

$$\begin{aligned} &\ln(\frac{P^{cd}_t}{P^c_t}) = \widehat{\xi}_2 + \widehat{\phi}_{22}\ln(\frac{P^{cd}_{t-1}}{P^c_{t-1}}) + \widehat{u}^{OLS}_{2t}, \\ &\text{and } \widehat{u}_{2t} \sim N(0, \widehat{\sigma^2_{u2}}). \end{aligned} \tag{29}$$

The values of the parameter estimates for this process $\widehat{\xi}_2$, $\widehat{\phi_{22}}$ and $\widehat{\sigma^2_{u2}}$ were imposed in a second stage where we estimated the values for the remaining parameters ($\phi_{11}, \xi_1, \sigma^2_{u1}$ and ω) by maximum likelihood within the state-space model. The admissible values of parameters were restricted as follows

$$\phi_{11} \in [0,1], \tag{30}$$

$$\omega \in [0,\infty], \tag{31}$$

$$\sigma^2_{u1} \in [0,\infty], \tag{32}$$

and

$$\xi_1 \in \left[0.01 + \{\ln(s_{cdt})\}_{12} - (1-\omega) * \left\{\ln(\frac{P^{cd}_t}{P^c_t})\right\}_{12}, -0.01 + \{\ln(s_{cdt})\}_{12} - (1-\omega) * \left\{\ln(\frac{P^{cd}_t}{P^c_t})\right\}_{12}\right]. \tag{33}$$

Restriction 30 ensures that this is a positive autocorrelated process. Restriction 31 keeps the elasticity of demand to its permissible range. $\{\ln(s_{cdt})\}_{12}$ and $\left\{\ln(\frac{P^{cd}_t}{P^c_t})\right\}_{12}$, are the mean values of the last three years of the estimation sample only. Hence restriction 33 implies that the initial value of the mean of ϑ would compensate for any systematic error in the recent residuals of the share demand equation. This mechanical rule incorporate the typical practice of extrapolating residuals to allow for possible structural breaks.

We estimated only the demand function for only one item in a two good system. The second equation would be redundant under the null that the price aggregator is correct.

Even if we avoided using tight theoretical assumptions when we constructed our data, they are still essentially purpose built. Hence we do not consider standard errors of the estimates or in sample goodness of fit to be a good indication of the reliability of the estimates in giving policy advice. And then, in practice, policy advice is predicated on models that accommodate for residuals; often exogenous time series models will be used to extrapolate residuals into the forecasts. Therefore what threatens policy advice is not the presence of a residual in the equation from which we estimate the key parameter se, but whether or not that residual is difficult to forecast. In this sense it matters not whether or not there is a large residual or even if it is trending, but only whether or not it is predictable.

To incorporate these insights, we assess our models in predicting the nominal shares over the last two years of quarterly data without using any of that data whatsoever in the estimation sample. Neither do we use the last two years' data on the relative prices; that series also has to be forecasted also within the state-space model. We assess the model on the basis of the Root Mean Squared Errors (RMSEs) in predicting the share series. In what follows, N is the estimation sample size comprising 50 quarterly observations.

Our data is plotted in Figure 1. If the relative prices are rising, then a fall in share would indicate the two components are complements and the estimate of the elasticity should pick this up. Note also that the estimate is judged by its ability to forecast the data on nominal shares to the right of the line. Immediately one can see that this is a serious challenge. Typically the share data is characterised by irregular cycles. This makes the trade off between anticipating a turning point or chasing the recent trend in the forecast period difficult.

5. Results

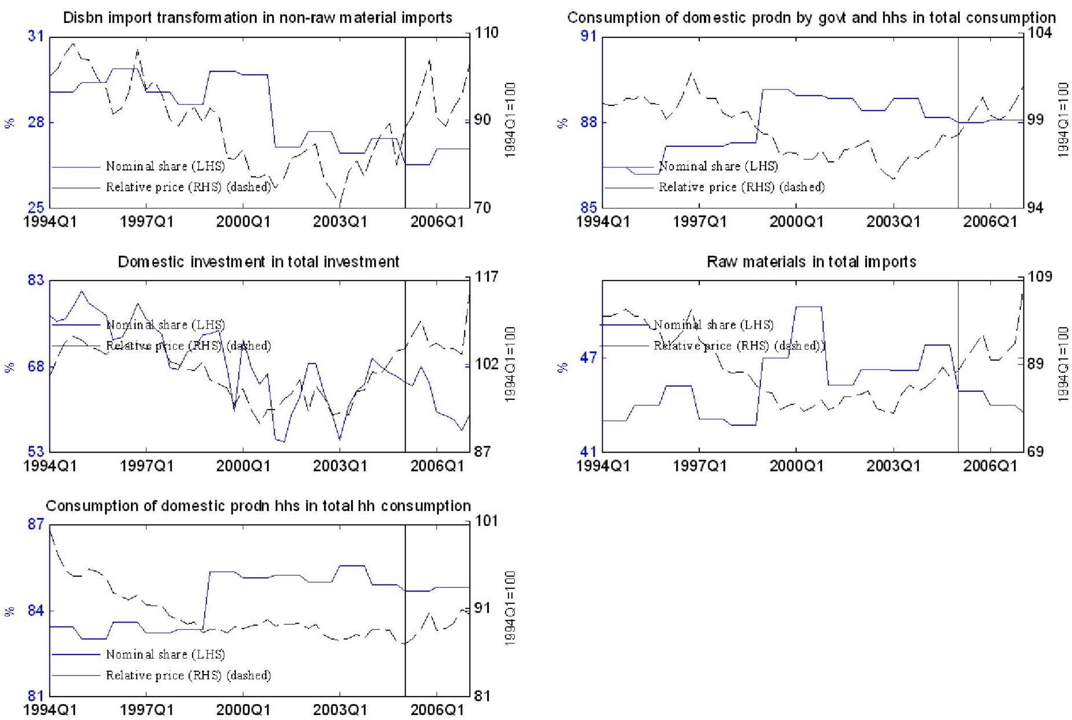
We can now turn to the parameter estimates, presented in Tables 3 to 7. The estimated elasticities of substitution (here all described as ω) as well as the other parameters provide rich information on Colombia's susceptibility to external shocks.

Table 3: Distribution in consumption and investment imports

	Max. likelihood est.
ϕ_{11}	0.13
$100^*\sigma_{11}$	5.21
$100^*\exp(\xi_1)$	28.72
ω	0.77

To begin with the estimated elasticity for the distribution sector indicates complementarity between domestic and imported items in the distribution transformation problem. The value

Figure 1: Data on five demand and supply relationships



of 0.77 could easily place it within the range of sensitivity identified in Section 2.1. And the long-run share, $\exp(\xi_1)$ capturing the average value of ι_{Tt} in equation 6, is estimated to be about 30%, implying that distribution is an important input into the final sale of imported consumption and investment goods.

On the other hand, we find that the elasticity of imports in consumption lies well above the Cobb Douglas restriction of one, indicating that there is not a great deal of sensitivity to external shocks for complementarity in arising from consumption. But as the effect of the elasticity on the real exchange rate is nonlinear in values of these elasticities around the Cobb Douglas value, the higher elasticity here might not offset of the influence of the distribution sector in exposing vulnerability.

But foreign and domestic investment are judged to be very strong complements; the data

Table 4: Domestic consumption of government and households

	Max. likelihood
	est.
ϕ_{11}	0.23
100*σ_{11}	0.93
100*exp(ξ_1)	87.6
ω	1.69

Table 5: Domestic investment

	Max. likelihood
	est.
ϕ_{11}	0.02
100*σ_{11}	9.24
100*exp(ξ_1)	64.24
ω	0.20

would have the elasticity of substitution close to the permissible lower bound of Leontieoff. This is some tentative evidence for a strong income effect associated with investment such that when either domestic or foreign investment becomes cheap, spending on both rises.

Then raw material imports are found to be close to Cobb Douglas substitutes to to consumption and investment items. In so far as imported raw materials require less domestic input before being transformed, this does not magnify the complementarity we highlighted in the distribution sector's role in importing consumer and investment goods.

Comparing Table 4 and Table 7, it appears that households are less likely to substitute domestic production for imports than the public sector. This might seem odd, bearing in mind that the government employs a larger proportion of domestic value-added factors of production than a typical tradable sector would do. But the governments consumption is different from its use of labour and capital inputs. In Colombia it is plausible that the government imports a large share of its consumption, for example, in defence. In any case, as a result, the private sector consumption elasticity is found to be close to the Cobb Douglas value.

In all cases the standard deviation of the unobserved component lie above zero. Thus the

Table 6: Raw materials in imports

	Max. likelihood est.
ϕ_{11}	0.03
100*σ_{11}	4.55
100*exp(ξ_1)	44.38
ω	1.23

Table 7: Domestic consumption of households

	Max. likelihood est.
ϕ_{11}	0.23
100*σ_{11}	1.09
100*exp(ξ_1)	82.24
ω	1.29

data favour time variation in ϑ over a fixed coefficient and vindicate our state space method. The estimated distributions of the parameter ϕ_{11} in each model are also revealing. In the case of investment especially this value is quite low meaning that there is very little information from past values that the Kalman Filter can use to build a forecast investment; the series is both volatile and not persistent.

The robustness of these estimates is tested by reporting on the forecast performance of the whole system. Table 8 reports the RMSEs from all five individual demand and supply relationships.

At first glance, the RMSEs in Table 8 all seem large. But these are forecasts made with our simple model. We would argue that the lowest RMSE here are consistent with what could be satisfactory performance when combined with off model judgement.

The RMSE in predicting distribution output share in the transformation of non raw material imports is larger at 8.2%. Given all the assumptions we had to employ to get this data, this is quite reassuring. We note however that the bootstrapped mean turned out to be much higher than the bootstrapped mode, indicating that there is a risk of some large errors.

Table 8: Estimates of the five demand relationships

RMSE estimates, calculated from the state-space.	
	Max. likelihood est.
Distribution in consumption and investment imports	8.20 (4.19)
Domestic consumption of hhds and gvt	1.07 (1.33)
Domestic investment	9.76 (21.41)
Raw materials in imports	5.28 (9.05)
Domestic consumption of households	1.87 (1.67)

* average over year one of the forecast (average over year two in brackets).

The low RMSE for consumption of about 1.07% indicates little risk of forecast error originating in the relations in the demand for domestically produced consumption for government and households together. While there is slightly more forecast error in the consumption problem just for households, the size of the error remains low enough not to cause alarm there either. The greater error might indicate either that the difference between the consumption deflator and the consumer price index brings with it some cost, or that our assumption that the price of domestic consumption is the same for government and for households.

But the greatest error by far is in the disaggregation of investment into its domestically produced and foreign produced components. The RMSE is 9.76% for the first year and then 21.41% by the second. Clearly the model is missing some of the cyclical behaviour in investment.

This could be because Colombian investment data is exceptionally volatile: the standard deviation of a detrended real investment series (including changes in inventories) is 18% compared to 8% for the same concept in UK data (both calculated on annual data from 1970-2007). This may be due to poor fixed investment data; it may be because there are irregular cycles in inventories; it may be because inventories are where the National Accounts office allocates its residual; it may also be due to aspects of tastes and technology not incorporated in our function form. For example the homothetic CES functional form may be restricting our investment model excessively.

Finally , we also note that the separation of raw materials from total imports does not involve too much unpredictability: the RMSE here is 5.28% for one year ahead.

6. Conclusions

Key policy decisions in open economies depend on values of parameters which capture the substitutability between tradable and nontradable sectors. We derived five important equations that straddle this split and compiled the data needed to estimate them for Colombia. A common feature of all equations, was that importing sectors were separated from domestic production, and for this reason, the database had to be purpose-built.

We estimated these equations. Our method takes account of prosaic adaptations that are made in using these parameters to give policy advice. Rather than using the theory of the model to derive data on the relative price series, we used more general index numbers. As these index numbers include the theoretical equations of the model as a special case, a residual would then appear in these key equations when the model and data were inconsistent. If that residual cannot be forecasted, it is likely that policy prescriptions based on these estimates would also prove unreliable.

Our estimates revealed that the contribution of the distribution sector in bringing imports to final consumers in Colombia is quite complementary to imports at the dock, with the estimates of the elasticity falling within the range of values that would imply that positive terms of trade shocks lead to large appreciations. Consumption imports appear to weak substitutes with domestic goods, favouring the opposite reaction. Both these estimates seemed to be relatively reliable. Given that the relation between these elasticities and the real exchange rate response is nonlinear, it might very well be that the distribution sector elasticity drives the overall response.

Estimates of imported investment indicated that it was a very strong complement with domestic investment in creating the capital stock of Colombia. But these estimates were also shown to be least reliable.

We can speculate as to how these important elasticities could evolve in the future. Broda & Weinstein (2006) observed that globalisation of the form of a greater vertical disintegration of production across national borders has lowered these elasticities for the United States. If this also holds true for developing countries, one might expect greater appreciations following terms of trade shocks in the future. On the other hand, improvements in the infrastructure (Colombia has a notoriously difficult terrain for importers) may act against this, by lowering importers' margins and raising the degree of substitutability. Our estimates suggest that monitoring these developments will be crucial in policy assessments of the responsiveness of the domestic economy to external shocks.

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