Health Expenditures and Externalities: Their Contribution to Economic Growth

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Abstract

This paper develops a dynamic, endogenous growth model that reveals the various pathways through which health expenditures, in the presence of an externality, augment labor effectiveness causing capital deepening and growth. In an inter-temporal environment, competitive firms employ capital and labor services, the latter is endogenously determined by households' inter-temporal choice to allocate some forgone consumption to health expenditures to augment own effective labor. These expenditures in turn lower harmful health externalities on other workers due to a lessening of the communicable nature of disease. The growth-health path ways suggest various potential points for policy interventions to ameliorate productivity, avoid a poverty-like trap and to enhance economic growth.

1 Introduction

This paper focuses on the relationship between health and economic growth in a low income country context. While it has long been recognized that income has a strong effect on the demand for health[1], in the early 1990s evidence began to accumulate on the positive effect of health on wages and productivity as discussed by Strauss and Thomas (1998). More recently, several studies, like Bloom et al. (2004), focus on the economy-wide growth effects of health and the pathways through which health affects economic growth[2]. In what follows, we identify three pathways seen in recent literature and combine them to culminate into a simple model of the relationship between health and growth. We believe that understanding these patheways can lead to better health policy for less developed countries.

First, Baldacci et al. (2004) explore the role of health expenditures, using a panel of 120 developing countries from 1975-2000. They find that spending on health within a period of time affects growth within that same period while lagged health expenditures appear to have no effect on growth, suggesting that the direct effect of health expenditure on growth is a *flow* and not a *stock* effect. They also find that health expenditures have indirect effects on growth via its positive effect on increasing physical capital investment.

Second, others have focused on health externalities (Azariadis, 1996; Gersovitz and Hammer, 2001), which can affect economic growth due to the contagious nature of many diseases and the link between own productivity on the productivity of coworkers. Health externalities are typically ignored when individuals make choices affecting their own health, which can lower the productivity of labor and dampen economic growth.

A third pathway is from poverty to health, which can produce a poverty-type trap (Sala-i-Martin, 2005). Low income limits the household's investment in health, precluding growth in labor productivity, thus decreasing incentives to save in an environment where foregoing consumption is already constrained by pressures to meet other basic needs. Slow growth in a country's stock of capital per worker in turn places downward pressures on growth in labor productivity which reinforces the poverty - growth cycle.

Missing from the literature is a growth model that combines the above pathways between health and growth. Understanding these health-growth linkages is important from a policy perspective for several reasons. First, labor services account for the largest share of gross domestic product (GDP) in most countries (Lucas, 1988, Mathew and Neumayer, 2006) which make improvements in the productivity of this resource a key component of

[1]See Lopez-Casasnovas, Rivera and Currais (2005) for a review of the earlier literature and a collection of new essays on health and economic growth.

[2]Spence and Lewis (2009) note that the methodological problems of capturing the effect of improved health on human capacity, income, welfare and linkages to economic growth require much more attention.

growth. Second, the infectious nature of disease implies the likelihood of market failure in the sense that individual expenditure on health does not take into account the benefits to other workers. Third, provisions for public investment in economy-wide infrastructure as well as local health clinics (including nutrition and sanitary conditions) are significant sources of growth. Fourth, from a broader policy perspective, a poverty-like trap can emerge where a low temporal household expenditures on health lead to unhealthily less productive workers which lowers wage income and health expenditures.

Our contribution is a parsimonious model that captures the linkages between health expenditures, labor productivity growth, and capital deepening in the presence of health externalities and a subsistence consumption constraint. We model an economy where households' have an incentive to forgo consumption in order to accumulate assets and to allocate health expenditures on health and supplemental nutrition which augments their own labor productivity. This household decision depend crucially upon the extent of heath externalities. Additional factors contributing to the household's decision and ultimately the economy's transition to the long run equilibrium are: the technology of firms, the structure of the economy as measured by its capital intensity, initial conditions (such as the country's stock of capital per worker relative to minimum consumption requirements), the household's endowed health technology, and the labor market's identification of individual worker productivity. We cast this household structure into a single sector Ramsey growth model and assess the model's health-growth linkages by calibrating the model to Sri Lanka data. In doing so, we show that the model provides a reasonable fit to the country's gross domestic product (GDP) and capital stock over the period 1990 to 2006.

We show that households are willing to forego some consumption to invest in health improvements that augment their labor productivity and consequently economic growth. Further, we find that growth is further augmented when more productive labor increases the productivity of capital which in turn incents an increase in capital deepening thus spurring economic growth. This points to the need for policy directed at increasing economic growth to consider actions that offer incentives for households to increase health expenditures. However, we also find that the externality causes private choices alone to lead to a non-optimal rate of economic growth. In the presence of a health externality, the more infectious are diseases, the greater the under-spending on own health, which can be partly overcome by public investment that makes more productive and/or lowers the cost of providing health care. Low income households struggling to meet subsistence consumption needs tend to benefit disproportionately from public investments that lowers the cost of health care because such investment increase their incentive to augment their health linked labor productivity, thus helping to pull them from the health-linked poverty trap.

The paper is organized as follows. The next section discusses the rational for key model primitives and relationships. Section 3 presents the theoretical model, defines and characterizes the equilibrium. Section 4 provides background to the economy of Sri Lanka and details the calibration procedure. In Section 5, we present the results of the model's empirical simulations. Section 6 concludes the paper.

2 Rational for key model primitives and links to the literature

We consider an environment in which households are endowed with a health technology that produces effective labor services (h) per unit of physical labor as a function of own health expenditures. Consistent with Baldacci et al. (2004), effective labor services are sustained and augmented on a temporal basis by expenditures on supplemental nutrition and health services such as antibiotics and sanitary conditions.[3] The availability of public and private facilities for preventative health care are taken as given but can be thought of as conditioning the efficiency of the technology. The opportunity cost of an incremental increase in health expenditures is foregone consumption and savings that could otherwise increase the household's stock of assets. If households are constrained by the need to meet minimum consumption requirements to survive, health expenditures can be entirely foregone.

Consistent with the epidemiology literature we also include a health externality as an argument in the production of h[4]. Only recently have economists attempted to incorporate epidemiological models into models of economic growth. Drawing on this literature, Roe and Smith (2008) link the epidemiological model of HIV/AIDs to a neoclassical growth model to study the effects of the disease on growth of the South African economy, while Anderson and May (1991) pursue a similar strategy but with a simpler form of disease transmission. In other work, Gersovitz and Hammer (2001) and Miguel and Kremer (2004) study the positive externalities associated with disease prevention. Our health technology approach is in the spirit of these contributions.

We consider a labor market environment in which firms recognize individual supply of effective labor services per unit of labor and remunerate workers accordingly. Firms behave

[3] We treat effective labor services (health) as a flow concept, instead of the human capital stock concept made famous by Grossman (1972), Lucas (1988) and Mankiw, Romer and Weil (1992). When labor quality at time t, $h(t)$, is determined by a given health endowment, but temporally affected by other nutrition and health services choices, the stock assumption can be avoided. Further, we abstract from aging.

[4] While consumption is also often included as an argument in the health production function, Strauss and Thomas (1998) find that beyond a minimum caloric intake level, consumption has little effect on labor productivity. Below, we capture this effect indirectly by positing a minimum consumption requirement in the household's felicity function which we interpret to be the minimum level to sustain basic health.

as if h_i for each worker i is known, which provides an incentive to spend on their own h_i each period. This behavior is consistent with microeconomic evidence that health expenditures are rewarded when workers are paid piece-rates (Foster and Rosenzweig, 1994).

To capture the effect of extreme poverty (low income) on health, we assume there exists a subsistence level of consumption below which a worker can not survive. This threshold can be interpreted as the level of consumption necessary to sustain the household's health endowment. To model this threshold in the simplest manner, we employ a Stone-Geary felicity function. The effect of a subsistence consumption requirement on transition growth can lead to a large share of households choosing negative savings and dis-accumulation of physical capital (Ben-David, 1998). The literature has also used subsistence consumption as a way to explain the empirical observation of a hump-shaped pattern of growth in models of endogenous economic growth (Easterly, 1994 and Steger, 2000).

Finally, effects of changes in the economic environment (including market forces and public health interventions) on households' investment in own human capital were emphasized years go by Rosenzweig (1988). Correa and Namkoon (1992) state more strongly those socioeconomic conditions are the main determinant of the health conditions of a population. Thus, the notion of a health-like production function that maps household choices into health outcomes conditional on other exogenous variables, such as the provision of public health facilities, has a long tradition in the literature.

3 Theoretical model

Our environment is a small, open and competitive economy. The economy is initially endowed with $\text{Ⱡ}(0)$ number of workers that increase at rate n. Each worker supplies the same quantity of hours, ℓ, to the labor market so that the total hours of labor supplied is $L(t) = \text{Ⱡ}(0)\, e^{nt}\ell$. Technology is represented by a neoclassical, constant returns to scale production function whose arguments are the stock of capital $K(t)$ and labor $L(t)$

$$Y(t) = F(K(t), L(t)(1 + h(t)))$$

where $h(t)$ is the effective labor service per worker.[5] In per worker terms we have:

$$y(t) = f(k(t), (1 + h(t)))$$

Effective labor services is produced by the household as function of own health expendi-

[5] We restrict $h \geq 0$, but recognize that $h > -1$ could also be considered.

tures and a health externality. For simplicity, we assume all workers are identical, but to capture the health externality, it is useful to identify the $i-th$ worker's $h_i\left(t\right)$ produced by the production function $\mathbf{h}:\mathbb{R}_+^2 \rightarrow \mathbb{R}_+$

$$\mathbf{h}^i\left(\varepsilon_i\left(t\right),\eta_i\left(t\right)\right) \tag{1}$$

with typical neoclassical properties of homogeneity, non-decreasing, strictly concave in arguments and everywhere continuous and twice differentiable. We perceive the production function (1) as a mapping of flows from expenditures $\varepsilon_i\left(t\right)$ on own health to effective labor services h per worker. We interpret $\varepsilon_i\left(t\right)$ to be expenditures on nutrition, medication and health services, and other lifestyle factors. The household chooses the allocation $\varepsilon_i\left(t\right)$ from total factor earnings (in units of $Y\left(t\right)$). The term $\eta_i\left(t\right)$ is a health externality.[6]

Given identical workers, effective labor status of the entire work force is the sum over all workers

$$\mathcal{H}\left(t\right)=\int_0^N \mathbf{h}^i\left(\varepsilon_i\left(t\right),\eta_i\left(t\right)\right)di = N\mathbf{h}\left(\varepsilon\left(t\right),\eta\left(t\right)\right)$$

where $N=\text{Ł}\left(0\right)e^{nt}$ is the total number of workers. Normalizing the number of initial workers $\text{Ł}\left(0\right)$ to unity, in per worker terms, we have

$$h\left(t\right)=\mathcal{H}\left(t\right)e^{-nt}=\mathbf{h}\left(\varepsilon\left(t\right),\eta\left(t\right)\right) \tag{2}$$

3.1 Health externality

To the capture the spirit of a health externality in the simplest terms, let the level of the externality η_i affecting worker i, be a non-decreasing, strictly concave, and everywhere continuous and twice differentiable function $\boldsymbol{\eta}$: $\mathbb{R}_+ \rightarrow \mathbb{R}_+$ of the health status h of another worker. Thus for two workers in the economy we have:

$$\eta=\boldsymbol{\eta}\left(h_i\right),\ \ \eta_i=\boldsymbol{\eta}^i\left(h\right)$$

Consequently, the health externality for one workers is the effective labor services of the other worker (i). Then, perform successive substitutions of η and h to obtain a composite function for one worker in the economy

$$h=\mathbf{h}\left(\varepsilon,\eta\right)=\mathbf{h}\left(\varepsilon,\boldsymbol{\eta}\left(h_i\right)\right)=\mathbf{h}\left(\varepsilon,\boldsymbol{\eta}\left(\mathbf{h}^i\left(\varepsilon_i,\eta_i\right)\right)\right)=\mathbf{h}\left(\varepsilon,\boldsymbol{\eta}\left(\mathbf{h}^i\left(\varepsilon_i,\boldsymbol{\eta}^i\left(h\right)\right)\right)\right)$$

[6] The production function can also be envisioned to embodied variables exogenous to the household, such as genetic endowments affecting agent's susceptibility to disease, and public sector factors such as the availability and quality of health services through local clinics.

We assume the composite function $h = \mathbf{h}\left(\varepsilon, \boldsymbol{\eta}\left(\mathbf{h}^{i}\left(\varepsilon_i, \boldsymbol{\eta}^{i}\left(h\right)\right)\right)\right)$ satisfies the inverse function theorem so that we can solve for h to obtain effective labor services for each worker as a function of own and other worker expenditures on health:[7]

$$h = \underline{\mathrm{h}}\left(\varepsilon, \varepsilon_i\right), \quad h_i = \underline{\mathrm{h}}^{i}\left(\varepsilon_i, \varepsilon\right) \tag{3}$$

The functions in (3) capture, through a complex contagion-like process, the effect of own and other worker health expenditures on the supply of effective labor services. For example, worker i purchases medication that cures own bacterial infection which increases own supply of labor services to the firm while also lowering exposure and severity of the disease for the other worker. The same is true for the other workers in the economy.

The nature of this type of externality allows for multiple assumptions about each worker's behavior. The most plausible behavior is for worker i to choose a level of expenditure ε_i in a way that only takes into account own effects of ε_i and ignores the contagion effect on the other worker. Thus the second argument in each of (3) is exogenous to each worker's decision resulting in:

$$h\left(\varepsilon\right) = \tilde{\mathbf{h}}\left(\varepsilon, \bar{\varepsilon}_i\right), \quad h\left(\varepsilon_i\right) = \tilde{\mathbf{h}}^{i}\left(\varepsilon_i, \bar{\varepsilon}\right) \tag{4}$$

At another, though far less plausible extreme, let all workers internalize the health externality, and in doing so let all believe that others spend at their level so that $\varepsilon = \varepsilon_i$ for all t. In this case, (3) reduce to[8]

$$h\left(\varepsilon\right) = \bar{\mathbf{h}}\left(\varepsilon\right), \quad h\left(\varepsilon_i\right) = \bar{\mathbf{h}}^{i}\left(\varepsilon_i\right) \tag{5}$$

In summary, we have for each worker

$$h\left(\varepsilon\left(t\right)\right) = \left\{ \begin{array}{ll} \tilde{\mathbf{h}}\left(\varepsilon, \bar{\varepsilon}_i\right) & \text{if worker ignores the health externality} \\ \bar{\mathbf{h}}\left(\varepsilon\right) & \text{if worker internalizes the health externality} \end{array} \right\} \tag{6}$$

The Cobb-Douglas specification used in the empirical model shows that the elasticity of $\bar{\mathbf{h}}\left(\varepsilon\right)$ with respect to ε is larger than the elasticity of $\tilde{\mathbf{h}}\left(\varepsilon, \bar{\varepsilon}_i\right)$ with respect to ε, implying that $\bar{\mathbf{h}}\left(\varepsilon\right)$ is the most efficient technology. When the worker employs production technology $\tilde{\mathbf{h}}\left(\varepsilon, \bar{\varepsilon}_i\right)$, h_i can be under produced.

[7] If we presume a Cobb-Douglas functional form $\left(h = a\varepsilon^{\beta} h_i^{\gamma}\right)$, the composite function is $h = h\left(\varepsilon, h_i\left(\varepsilon_i, h\right)^{\gamma}\right) = a\varepsilon^{\beta}\left(a_i \varepsilon_i^{\beta} h^{\gamma}\right)^{\gamma}$ where a_i and a are scale parameters and β and γ are positive fractions. Solving for h results in: $h = \left(\alpha a_i^{\gamma} \varepsilon^{\beta} \varepsilon_i^{\gamma\beta}\right)^{\frac{1}{1-\gamma^2}}$.

[8] To illustrate using $h = a\varepsilon^{\beta} h_i^{\gamma}$, and assuming identical workers such that $i = j$ equation (4) become $h = a\varepsilon^{\beta} A$, where $A = a_{i=j}^{\frac{\gamma}{1-\gamma}} \varepsilon_{i=j}^{\frac{\beta\gamma}{1-\gamma}}$ and equation (5) becomes $h_i = a^{\frac{1}{1-\gamma}} \varepsilon^{\frac{\beta}{1-\gamma}}$.

3.2 Household behavior

Households maximize the discounted present value of utility from consumption and save subject to their earnings from labor and returns to assets, minus spending on health. The household's felicity function reflects the fact that households must overcome a subsistence level of consumption needed to sustain a health endowment, below which the worker cannot survive. The roots of this formulation comes from the literature that inter-temporal utility should include an inherited stock of human health (Grossman, 1972).[9]

Let ω^o be the worker's endowed health status and $c(t)$ be per worker consumption. Then we assume there exists a threshold level of consumption c^o, below which a worker cannot survive. For consumption levels c above c^o, health status defaults to the worker's endowed health status ω^o. For simplicity, we choose the Stone-Geary form with constant inter-temporal elasticity of substitution (CIES) given by θ:

$$\frac{(c-c^o)^{1-\theta}-1}{1-\theta} : \quad 0<\theta\neq 1$$

3.2.1 Household budget constraint

At every point in time, the representative household provides per worker labor services $(1+h(t))\ell$, in exchange for wages $w(t)$. Households own assets A can be rented out as capital to firms at rate r^k or loaned to other households in return for interest income $r = r^k-\delta$. Since foreign liabilities are not allowed, domestic assets are equivalent to domestic capital K. The household allocates income to purchases of an aggregate consumption good and expenditures on health services. The per worker household budget constraint for the representative household is (omitting the t notation):

$$\dot{k} = w(1+h(\varepsilon)) + k\left(r^k-\delta-n\right) - c - \varepsilon \tag{7}$$

where $h(\varepsilon)$ is taken from (6), $k = K/e^{nt}\ell$, and labor is remunerated for the effective labor services $(1+h(\varepsilon))$ supplied.

3.2.2 The household's intra-temporal problem

The intra-temporal decision is to choose the level of health expenditures, $\varepsilon(t)$, per unit of labor to maximize returns to expenditures ε on effective labor augmentation h for each t. Given $h(\varepsilon(t))$ from (6), the problem is

[9] See examples by Glewwe and Miguel (2008) and Rosenzweig and Schultz (1983).

$$
\begin{array}{ll}
\underset{\{\varepsilon(t)\}}{Max} & w(t)(1+h(\varepsilon(t)))-\varepsilon(t) \\
\text{s.t.} & \varepsilon(0)\geq 0
\end{array}
\tag{8}
$$

Assuming an interior solution, we obtain from the first order condition

$$
wh_{\varepsilon}(\varepsilon(t))-1=0
$$

from which we obtain the decision rule for the optimal intra-temporal level of health expenditures as a function of w

$$
\varepsilon=\varepsilon(w) \tag{9}
$$

Given the properties of $h(\varepsilon)$, it is easily shown that $\varepsilon(w)$ is an increasing function of w. Substituting the decision rule (9) into $h(\varepsilon)$ in (6) gives the supply of effective labor in dual form as

$$
\mathbf{h}(w)=h(\varepsilon(w)) \tag{10}
$$

The household's (dual) indirect function for returns to labor can be expressed as

$$
\mathbf{\Pi}(w)=w(1+\mathbf{h}(w))-\varepsilon(w)
$$

It is easily shown that the envelope theorem implies that the supply of effective labor services is simply the gradient of $\mathbf{\Pi}(w)$

$$
\frac{\partial\mathbf{\Pi}}{\partial w}=\mathbf{h}(w) \tag{11}
$$

3.2.3 The household's inter-temporal problem

The household maximizes the inter-temporal, dynastic utility function. Using the intra-temporal results (9) and (10), the representative household problem is:

$$
\begin{array}{ll}
\underset{\{k(t),c(t)\}}{Max} & U=\int_0^\infty \frac{(c(t)-c^o)^{1-\theta}-1}{1-\theta}e^{(n-\rho)t}dt \\
\text{s.t.} & \dot{k}(t)=w(t)(1+\mathbf{h}(w(t)))+k(t)\left(r^k(t)-\delta-n\right)-c(t)-\varepsilon(w(t)) \\
& k(0),\ c(0)\geq c^o \\
& \lim\limits_{t\to\infty}\left\{k(t)\cdot\exp\left[-\int\limits_0^t [r(v)-n]\,dv\right]\right\}\geq 0
\end{array}
\tag{12}
$$

Here the household chooses $k(t)$ and $c(t)$ to maximize the discounted present value of utility, subject to the flow budget constraint, the stock of initial capital $k(0)$ and consumption $c(0)$, and the transversality condition. From this problem we obtain the basic Euler condition for choosing consumption per worker over time

$$\frac{\dot{c}}{c-c^{o}}=\frac{1}{\theta}(r^{k}-\rho-\delta) \tag{13}$$

3.3 Firm behavior

Firms are assumed to recognize $\mathbf{h}(w)$ provided by each worker. The firm takes the prices of output, and the rental rates of labor and and capital as given to maximizes intra-temporal profits. Using (10), the representative firm's optimization problem is, in per worker terms:

$$\underset{K(t),L(t)}{Max}\left\{f\left(k\left(t\right),\left(1+h\left(w\left(t\right)\right)\right)\right)-w\left(t\right)\left(1+h\left(w\left(t\right)\right)\right)-r^{k}\left(t\right)k\left(t\right)\right\} \tag{14}$$

At each t firms maximize profits by equating the marginal product of capital to the rental price and the marginal product of labor to the wage rate. The result is two conditions in three endogenous variables $\left(w,r^{k},k\right)$ that must hold at each t

$$\begin{aligned}\mathbf{R}\left(k,w\right) &\equiv r^{k}=f_{k}\left(k,\left(1+h\left(w\right)\right)\right) && (15)\\ \pi\left(k,w\right) &\equiv f\left(k,\left(1+h\left(w\right)\right)\right)-w\left(1+h\left(w\right)\right)-\mathbf{R}\left(k,w\right)k=0 && (16)\end{aligned}$$

3.4 Definition and characterization equilibrium

Given initial economy-wide endowments $\{K(0),L(0)\}$, a competitive equilibrium is a sequence of household allocations $\{c(t),\varepsilon(t)\}_{t\epsilon[0,\infty)}$ that satisfy the household's intra-temporal (8) and the inter-temporal (12) problem, firm allocations $\{K(t),L(t)\}_{t\epsilon[0,\infty)}$ that satisfy the firm's profit maximization problem (14), and market clearing conditions for capital, labor and the economy-wide final good yielding positive resource rents $\left\{w(t),r^{k}(t)\right\}_{t\epsilon[0,\infty)}$. Equilibrium at each t leads to the economy-wide identity

$$y=w\left(1+h\left(\varepsilon\right)\right)+r^{k}k=c+\varepsilon+\dot{k}+k\left(r^{k}-\delta-n\right) \tag{17}$$

where the equilibrium value of output per worker y equals factor payments which equal total expenditures, including expenditures on health, plus savings per worker.

The intra-temporal equilibrium results in the firm's equilibrium conditions (15) and (16). The inter-temporal equilibrium comprises a system of three differential equations in three unknowns. Using (15) the household budget constraint becomes

$$\dot{k}=\mathbf{K}\left(k,w,c\right)\equiv w\left(1+h\left(w\right)\right)+k\left(\mathbf{R}\left(k,w\right)-\delta-n\right)-c-\varepsilon\left(w\right) \tag{18}$$

Two additional equations are needed, one of which is the Euler equation

$$\dot{c} = \frac{c - c^o}{\theta}(\mathbf{R}(k, w) - \rho - \delta) \tag{19}$$

The next step is to derive the differential equation for w using the zero profit condition for firms (16). Differentiating this function with respect to time

$$\pi_k(k, w)\dot{k} + \pi_w(k, w)\dot{w} = 0$$

and substituting (18) for $\dot{k}$, and solving for $\dot{w}$ we obtain the third differential equation to complete the system

$$\dot{w} = \mathbf{W}(k, w, c) \equiv -\frac{\pi_k(k, w)}{\pi_w(k, w)}\mathbf{K}(k, w, c) \tag{20}$$

We now have a square system of three autonomous differential equations (18), (19) and (20) in three unknowns k, w and c. A solution $\{k, w, c\}_{t\epsilon[0,\infty)}$ permits the calculation of the remaining endogenous variables $\left\{r^k, c, h, \varepsilon\right\}_{t\epsilon[0,\infty)}$. An empirical solution can be obtained by finding values $\{k^{ss}, w^{ss}, c^{ss}\}$ satisfying the three differential equations for $\dot{k} = \dot{w} = \dot{c} = 0$. Once these values are known, the steady state values for the remaining variables are calculated.[10] To empirically solve for the transition path to the steady state we follow Brunner and Strulik's (2002) method to solve the system.

4 Application of the model: Sri Lanka

To assess the model's health-growth linkages we calibrate the model to data since the equations of motion are not analytically tractable. The application to Sri Lanka is relevant from a policy perspective as the country has expanded health care coverage since the 1930s, including to the rural poor (Rannan-Eliya and Sikurajapathy, 2008). This raises questions of how health care policy can be improved for poor and middle income countries with an existing health care system. Further, Sri Lanka faces significant socioeconomic challenges of poverty and malnutrition. Since 1990, the poverty head count ratio has fallen only slightly to just under 25% (Sir Lanka 2004a). Over a fifth of the adult population remains undernourished while a third of children are malnourished (WDI). Most households spend a large share of total expenditures to sustain life; 50-70% of the population spends an average of 44.5% of total household expenditures on food (Sri Lanka, 2004a). Further, daily caloric

[10]For details of an analytical solution to the steady state equilibrium and an analysis of the steady state see Wisniewski (2008).

consumption averages only 2390 calories (FAO Statistical Yearbook, 2004), placing households relatively close to their subsistence level of caloric consumption of 2030 (Sri Lanka, 2004b). In the Stone-Geary framework, this implies that total consumption is only 18% above the parameter c^o. The closer consumption is to c^o, the less incentive a household has to forego consumption for *both* savings that increase the country's capital stock and expenditures on health that increase the household labor productivity.

4.1 Empirical estimation and calibration

In fitting the model to data, we recognized that real economies entail many sources of exogenous technological change. Thus we include an exogenous Harrod rate of total factor productivity $x \geq 0$. To validate the model's ability to predict, we fit the model to the 1990 point on the country's growth path. The aggregate production function is assumed Cobb-Douglas

$$Y = AK^{\alpha}\left(L\left(1+h\right)e^{xt}\right)^{1-\alpha} \tag{21}$$

where A is a scaling parameter and $0 < \alpha < 1$. Competition assures inputs are paid their marginal products so that equilibrium values imply

$$Y = r^k K + wL\left(1+h\right)e^{xt}$$

with factor share $\alpha = r^k K/Y$. The determination of labor's share of national income $(1-\alpha)$ is adjusted to include estimates of employee compensation to account for self employment, often recorded as operating surplus of unincorporated enterprises (OSPUE) in national accounts data (Gollin, 2002). The value, 0.76, is chosen based on Duma's (2007) study of growth contributions for the case Sir Lanka. This value closely approximates the average across 31 countries in Gollin (2002).

The key differential equations of the empirical model (18), (19) and (20) , written in terms of effective units of labor become:

$$\begin{aligned}
\dot{\hat{c}} &= \frac{\left(\hat{c}-c^o\right)e^{-xt}}{\theta}\left(\mathbf{R}\left(\hat{k},\hat{w}\right)-\rho-\delta-\theta x\right) && (22)\\
\dot{\hat{k}} &= \hat{w}\left(1+\mathbf{h}\left(\hat{w}\right)\right)+\hat{k}\left(\mathbf{R}\left(\hat{k},\hat{w}\right)-\delta-n-x\right)-\hat{c}-\hat{\mathbf{e}}\left(\hat{w}\right) && (23)\\
\dot{\hat{w}} &= -\frac{\partial\hat{\pi}/\partial\hat{k}}{\partial\hat{\pi}/\partial\hat{w}}\mathbf{K}\left(\hat{k},\hat{w},\hat{c},t\right) && (24)
\end{aligned}$$

where $\hat{k} = ke^{-xt}$, $\hat{w} = we^{-xt}$, and $\hat{c}\left(t\right) = ce^{-xt}$.

Population growth (n) is taken as the average rate over the 1990-2006 period (WDI). Parameters for the time rate of preference ($\rho = .02$), the depreciation rate ($\delta = .03$) and the inter-temporal elasticity of substitution ($\theta = 2$) are taken in accordance to a general convention with other growth models (Barro and Sala-i-Martin, 2004; Stokey and Rebelo, 1995). Data from World Development Indicators and the perpetual inventory method was used to estimate capital stock ($K(0)$), initial output ($Y(0)$), and labor ($L(0)$).

In principle, the Solow residual from which we obtain the Harrod rate of total factor productivity should be purged of the health effects the model is designed to produce. We draw upon the contribution of Cole and Neumayer (2006) who find that across 52 countries, a 1% increase in the prevalence of undernourishment, malaria and lack of access to safe water each results in a .21-.33%, a 1.06% and a .63% percent decrease in TFP, respectively. Effectively, this gives both an estimate of the contribution of health to TFP as well as an estimate of the population health endowment, μ. Thus, the total contribution of μ to TFP ranges from $.019 - .0202$. Taking the lower bound, $x_{adj} = x - \mu = 0.001$.

The notion of an empirical health production function is common to the health literature, however this phase of calibration is a more subjective process. The initial value for labor augmentation $h(0)$ includes the estimate of health endowment, μ. Initial health expenditures are taken to be 2% of GDP, such that initial health expenditures are $\varepsilon(0) = .02Y(0)$[11]. Then, initial wages are computed as:

$$w(0) = \frac{Y(0) - \alpha Y(0)}{1 + h(0)}$$

And the share of $w(0)$ due to health expenditures is computed as:

$$\beta = \frac{\varepsilon(0)}{w(0)\,h(0)}$$

To the best of our knowledge, there are no empirical estimates of γ; this is taken to be 0.60.[12] The value of the productivity parameters, A and a, are adjusted to satisfy equilibrium relationships. For the Stone-Geary felicity function, the subsistence level of consumption (c^o) is derived from the official poverty line reported by Department of Census and Statistics of Sri Lanka (2004).[13] Table 1 in Appendix B summarizes the parameter and initial values employed in the model.

Given the steady state variables, the Brunner and Strulik (2002) method of backward

[11] In Sri Lanka private and public health expenditures as a percent of GDP were 2% each (WDI, 2001-2005). Since we are interested in the household's expenditures on health, we chose 2%.

[12] A range of values for γ were used in the empiracle estimation of the models.

[13] See Wisniewski (2008) for details on poverty line calculations.

integration is used to solve the non-autonomous system of differential equations (22) , (23) and (24) to obtain the transition path of $\left(\hat{c}(t), \hat{k}(t), \hat{w}(t)\right)$ for $t \in [0, \infty)$. The system becomes non-autonomous when we normalize by e^{xt} in the presence of the subsistence consumption parameter c^o which, for $x > 0$, becomes $c^o e^{-xt}$. The basic idea of the Brunner Strulik method is to trace the solution of (23) from a value in an $\varepsilon-$neighborhood of the steady state back to the initial condition $\left(\hat{k}(0)\right)$. Then the trajectory is transformed back into forward looking time by a second time reversal.

5 Empirical Analysis

We validate the model over the 1990 to 2005 period by measuring the model's predictions of GDP and capital stock to corresponding time series data on the same variables using four different measures including Theil's U statistic and the concordance correlation coefficient. Finding that the model fits the data reasonably well, we then perform a number of simulations to assess the sensitivity of the model to various parameterization. The model results show that both model's prediction of capital stock and GDP track the data over the 1990-2006 period surprisingly well. Measures of the model's forecast errors for Y and K appear in Table 2 in Appendix B. These results lend credibility to the notion that the model is capturing some of the complex economic forces linking individual behavior, health expenditures and economic growth. The presentation of our results illustrates the linkages between health expenditures and health externalities found in our model and their contribution to economic growth. The figures and tables discussed below are located in Appendix A and B, respectively.

5.1 Direct impacts of health on growth

We begin by showing that health expenditures *directly* affect the growth in effective labor services, which *directly* affect the growth in output. Figures 1 and 2 illustrate the half life of the growth path to the steady state for expenditures on health and effective labor services, demonstrating that the health externality has a level effect on the steady state outcomes. When workers ignore the health externality, there is an under supply of steady state expenditures on health (roughly 2% of GDP throughout the transition path) compared to when the externality is internalized (health expenditures are roughly 8% of GDP).[14] This results in a lower level of effective labor supplied to the market at every point in time. This in turn, pulls down output per worker along the transition path shown in Figure 3. Thus,

[14]In terms of constant 2000 US dollars, health expenditures per worker are \$50 and \$177 at the half life to the steady state when the externality is ignored and internalized, respectively.

our model exhibits features of endogenous growth whereby competitive markets alone do not yield the optimal transition path of the economy to the long-run equilibrium.

Second, the steady state (long-run) results presented in Table 3 show that effective labor-output ratio is higher when the health externality is internalized, resulting in a higher capital-output ratio and a higher labor income-output ratio. While annual wage income per unit of labor ($w_{ss}(1+h_{ss})$) is higher, the wage paid by firms for effective labor service per unit of gross output is lower, thus yielding benefits to both the household and firms. Steady State GDP is about 6 percent higher, while the consumption-output ratio is lower. This latter result implies that a relatively larger share of gross output is allocated to health expenditures when the health externality is internalized causing the ratio of consumption to GDP to also be higher.[15] These two findings suggest that policies directed at growth should consider public health subsidies to correct for market failures in the market for health goods (including spending on health services and better nutrition).

Third, to provide insights into the pathways through which effective labor (h) *directly* effects the economy during transition growth we conducted a growth accounting exercise of the model results using the aggregate production function in which we decompose the growth of output per worker ($\dot{y}/y$) into the growth of capital per worker $\left(\dot{k}/k\right)$, effective labor $\left(\dot{h}/(1+h)\right)$ and TFP (x).[16] These results are presented in Table 5, Column 4. They show that for the 1990-2010 period, the *direct* effect of growth in h on growth on output per worker is over 7% when the health externality is internalized and over 6% when it is ignored[17] Thus in transition, capital has a much larger *direct* effect on the growth in output than does h since the growth in k is greater than both x and $\dot{h}/(1+h)$, (Column 5-7). However, h also has an *indirect* effect on y, as it operates through k.

5.2 Indirect impacts of health on growth

Health expenditures used to produce effective labor (h) *indirectly* affect the growth of output y and household income in two ways. One, an increase in the supply of h increases the marginal product of k thus, through returns to capital r, induces households to increase their total savings allocated to increasing their asset stock, k. To see this *indirect* effect, we conducted an empirical simulation in which we don't allow for h to grow ($h(t)=0,$ for all t) so that growth in k is the only endogenous variable that affects growth in y. Clearly, Table

[15] Our results are likely to under estimate the health externality effects since the data needed to properly estimate γ are unavailable. This suggests the need for future work

[16] See Wisniewski (2006) for details of the growth accounting exercise.

[17] The balanced growth feature of the model causes the steady state values of Y and K to grow at the exogenous rate of $x+n$, w grows at rate x and h grows at rate zero.

6 shows that when growth in h is eliminated, the growth in y is only 85-99% of the base growth rates presented in Table 5. These lower rates of growth are due to the now slower growth in capital stock per worker. Thus, when h is allowed to grow, the growth effects of k on y are larger and more persistent over time. Thus an increase in the supply of effective labor increases the marginal productivity of a given level of capital stock, of which the higher remuneration to households provides an incentive to increased the level of saving and hence level of assets.

Two, an increase in the supply of h also increases labor income, which the household can allocate toward incremental increases in consumption, savings and additional health expenditures. The growth in capital stock further augments the marginal productivity of labor, which increases remuneration to labor, as well as providing incentives for the household to increase health expenditures, which further increases the supply of effective labor to the market. Thus h increases both capital's contribution to growth and to labor income, which further increases the productivity of labor services. These effects are strengthened when the worker internalizes the health externality. From a policy perspective, one can not ignore the direct and indirect linkages between health expenditures (and thus labor productivity) and growth. This is significant given that labor services is the largest share of GDP for most countries.

5.3 Poverty-like traps

In Section 5.2 we showed that when the health externality is ignored, labor income is lower. The presence of a minimum consumption requirement necessary to sustain life causes households living close to this constraint to forego saving and health expenditures relative to wealthier households. Lowering their ability to spend on health, tends to "flatten" economic growth resulting in a poverty-trap-like outcome. We use the Stone-Geary framework to show that this type of poverty trap is possible by conducting a simulation in which we place the representative household closer to their subsistence consumption level by increasing c^o.[18] Figure 4 shows that output per worker requires four times the half life to reach the steady state as the base model. Intuitively households' have less incentives to forgo consumption in the short and intermediate run, thus slowing savings growth and consequently growth in health expenditures and asset accumulation. Capital deepening is slowed as is the growth in labor productivity, since the direct and indirect sources of growth previously discussed are dampened in the short and intermediate run. This dampening causes a "flatter" transition path to the steady state than the less constrained path of the base solution. Growth accel-

[18] To illustrate this result, we increased c^o to $\bar{c}^o = c^o\,(50)$, set the rate of exogenous technological change to zero and limit the analysis to the case where households do not account for the health externality.

erates as consumption rises above subsistence, allowing savings to rise and capital stock to accumulate. Capital accumulation increases labor productivity resulting in an increase in the household's incentives to spend on health to further augment their own labor productivity. Once households are considerably above their subsistence consumption level, the growth in capital falls as the marginal product of capital and the marginal product of own augmentation to labor effectiveness declines. This is an example of the hump-shaped pattern of growth discussed by Easterly (1994). For economies with income constrained households, programs to increase the quality and availability of health care should help to lessen the poverty-like health trap and accelerate economic growth.

5.4 Productivity multiplier impacts on growth

Finally, we show how the direct and indirect effects of h on growth can be magnified by multiplier effects. Public investments in roads, infrastructure and institutions that increase the productivity of firms can be captured by an increase in the scale parameter, A, of the economy-wide production function in equation (21). In our model, this effect increases the productivity of capital and labor, thus inducing households to increase their labor effectiveness, causing output to increase. Further, a positive shock to the scale parameter a of the effective labor production function h due, for example, to public improvement in quality and quantity of health clinics also has a multiplier like effect that is not present in the typical growth model.[19] In Table 4 we show the results of an empirical simulation we increased each scale parameter by 10%, and report the steady state values for key variables. The results show that a multiplier effect of both a and A is significant in both cases. This gives the gross returns from the change in A and a, and suggests how this framework could be used to guide public investment decisions.

6 Conclusions

Increasing labor productivity is a top priority in most countries because labor services comprise the largest share of GDP and increasing labor productivity tends to stimulates savings and capital deepening which spurs economic growth. Our contribution is the development of a structural dynamic model that shows the various pathways through which household's incentives to forego consumption in order to spend on own health augments labor productivity which both directly and indirectly speeds up the rate of capital deepening which further incents households to increase their health expenditures. Capital deepening increases la-

[19] See Footnote 7 for the functional form of the the effective labor production function.

bor productivity which in turn incents health expenditures leading to a more sustained and persistent economic growth over time. We show the incentive to spend on health depends crucially upon the extent of heath externalities associated with the communicable nature of many diseases. Indeed, the health externality causes private choices alone to lead to a non-optimal rate of economic growth. Additional factors that contribute to household decision over health expenditures and ultimately to the economy's transition to the long run equilibrium are: the technology of firms, the structure of the economy as measured by its capital intensity, initial conditions (such as the country's stock of capital per worker relative to minimum consumption requirements), the household's health technology, and the labor market's identification of individual worker effectiveness.

We highlight a few of the important policy implications of our results. First, since health technology arises out of capital accumulation (i.e. facilities for preventative health care, availability of antibiotics, clean water, and sewer water systems), the rise in capital reinforces the incentive to allocate resources toward health expenditures. This finding suggests that countries like Sri Lanka may benefit from an increased allocation of resources toward health expenditures where malnutrition and poor health remain a significant problem. Encouraging and/or subsidizing expenditures on supplemental food and nutrition can result in poverty alleviation in countries where at least a quarter of the population lives below the national poverty line.

Second, while a government supported health awareness campaign is a direct approach to addressing the health externality, an indirect approach is interventions to increase the ability of households to gain access to the health services, the provision of health information, providing free or low cost immunization services and so on. The model suggests that sustaining labor productivity through spending on health might be a key to maintaining the persistence in the provision of effective labor services and hence maintaining the persistence of economic growth. Consequently, countries should also maintain their persistence in the provision of medical services and facilities that incent households to maintain their health expenditures, even in the presence of negative economic shocks.

Third, the model allows for the opportunity to study how a country might more quickly transition out of a poverty-like trap. Low income households living close to their basic consumption needs have little ability to save, thus lowering the growth of a country's physical capital stock, which lowers growth in labor productivity, which further lowers incentives to invest in health. Ignoring the externality in this case is particularly harmful for economies with large impoverished populations. International donors to assist government in the provision of health services to fill gaps that cannot otherwise be met by domestic resources alone could be one of the keys to alleviating the poverty trap.

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A Results figures showing key variables along the transition path

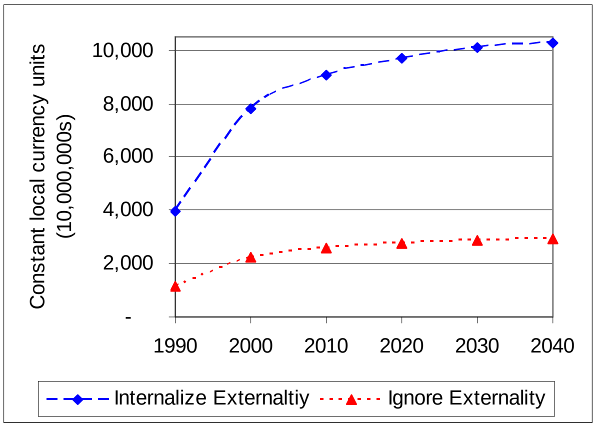

Figure 1: Health expenditures

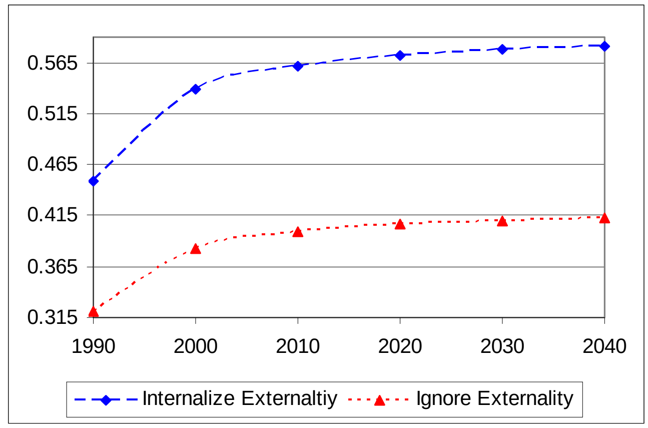

Figure 2: Effective labor services (index)

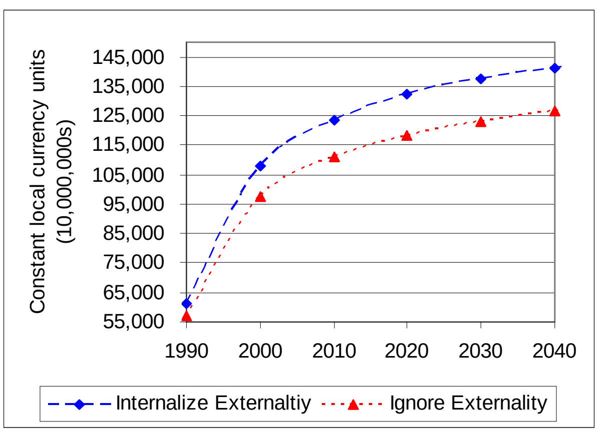

Figure 3: Per worker output

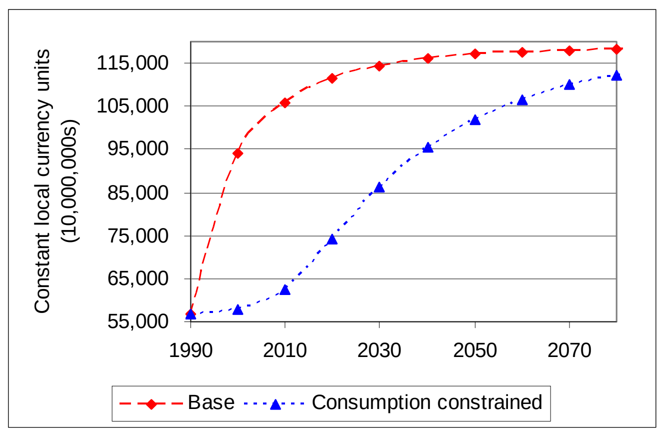

Figure 4: Per worker output: comparing results for c^o vs $\bar{c}^o$

B Results tables

Table 1: Parameter values and variable initial values

Parameter Values			
$\alpha = .24$	$\rho = .02$	$\beta = .11$	$x_{adj} = x - \mu = .001$
$n = .013$	$\delta = .03$	$x = .02$	$A = 3,925.95$
$\theta = 2$	$\gamma = .60$	$\mu = .019$	$a = .5$

Variable Initial Values (1990)		
$Y(0) = 56,899.7$ LCU	$w(0) = 32,785.3$ LCU	Ł $(0) = .98$
$K(0) = 28,669$ LCU	$r(0) = .48$ LCU	$c^0 = 1,077.7$ LCU
$C(0) = 46,657.8$ LCU	$\varepsilon(0) = 1,137.99$ LCU	

Note: For convenience, the initial values are scaled down by a factor of 10,000,000

Table 2: Various measures of model fit: 1990 to 2005

	GDP	Capital Stock (K)
Pearson's Correlation Coefficient	0.973	0.992
Concordance Correlation Coefficient	0.776	0.904
Theil's U Statistic	0.133	0.135
Mean Absolute Error	21.45	15.678

Concordance Correlation Coefficient: $2S_{12}/(S_1^2 + S_2^2 + \left(\bar{Y}_1 - \bar{Y}_2\right)^2)$

Theil's U Statistic: $\sqrt{\sum_t \left(Y_t - \hat{Y}_t\right)^2 / \sum_t Y_t^2}$

Mean Absolute Error: $\left(\sum_t \frac{|Y_t - \hat{Y}_t|}{Y_t}/N\right) 100$, N equals number of observations

Table 3: Steady state ratios, comparison of internalize to ignore the health externality

	Ratio
GDP	1.060
Gross Output (Y)	1.120
Consumption Output Ratio	.985
Effective Labor Output Ratio	1.335
Capital Output Ratio	1.057
Labor income Output Ratio	1.051
Wage Output Ratio	0.944

Output ratios are calculated in GDP per effective worker terms.
Thus in the table we have: $\frac{k(t)/GDP(t)^{internalize}}{k(t)/GDP(t)^{ignore}}$

Table 4: Steady state results showing the ratio of an increased multiplier to the base case

	% of Base Solution		
	$(1.10 \cdot a)$	$(1.10 \cdot A)$	$(1.10 \cdot a)$ and $(1.10 \cdot A)$
Internalize Health Externality			
GDP	112.3	115.1	129.6
Gross Output (Y)	114.3	115.4	132.3
Consumption to Output Ratio	99.5	99.9	99.4
Effective labor to Output Ratio	124.3	91.5	112.8
Capital to Output Ratio	101.8	100.2	102.1
Labor Income to Output Ratio	101.8	100.2	102.1
Wage to Output Ratio	89.1	98.5	87.4
Ignore Health Externality			
GDP	110.7	103.8	127.5
Gross Output (Y)	111.3	103.2	128.4
Consumption to Output Ratio	99.8	100.1	99.8
Effective Labor to Output Ratio	125.5	72.7	114.0
Capital to Output Ratio	100.6	99.5	100.7
Labor Income to Output Ratio	90.4	109.2	88.9

Table 5: Growth accounting results for output per worker, y

Year	$\dot{y}/y$	Contributions from			$\dot{k}/k$	$\dot{h}/(1+h)$
		$\dot{k}/k$	$\dot{h}/(1+h)$	x		
Internalize Health Externality						
1990	0.194	0.179	0.015	0.00076	0.741	0.064
		91.9%	7.7%	0.4%		
2000	0.022	0.020	0.002	0.00076	0.078	0.007
		88.4%	7.7%	3.4%		
2010	0.010	0.009	0.001	0.00076	0.032	0.003
		85.4%	7.0%	7.6%		
2020	0.006	0.005	0.0004	0.00076	0.017	0.001
		80.7%	6.7%	12.7%		
2030	0.004	0.003	0.0002	0.00076	0.010	0.0007
		77.1%	4.8%	18.1%		
2040	0.003	0.002	0.0001	0.00076	0.007	0.0004
		73.1%	3.1%	23.8%		
Ignore Health Externality						
1990	0.178	0.166	0.011	0.00076	0.688	0.0594
		93.4%	6.2%	0.4%		
2000	0.021	0.019	0.001	0.00076	0.0755	0.0065
		89.9%	6.6%	3.6%		
2010	0.010	0.008	0.0006	0.00076	0.0306	0.0026
		86.9%	6.3%	7.9%		
2020	0.006	0.005	0.0003	0.00076	0.0157	0.0013
		81.7%	5.2%	13.1%		
2030	0.004	0.003	0.0002	0.00076	0.0089	0.0007
		76.6%	4.9%	18.5%		
2040	0.003	0.002	0.0001	0.00076	0.0054	0.0004
		75.5%	3.2%	24.5%		

Table 6: Growth of output per worker and and capital per worker; Model simulations with no growth in h

Year	$\dot{y}/y$	% of Base Solution		$\dot{k}/k$	% of Base Solution	
	$(h(t)=0)$	Internalize	Ignore	$(h(t)=0)$	Internalize	Ignore
1990	0.169	85.2	95.0	0.698	93.8	101.4
2000	0.019	85.9	90.0	0.073	93.3	97.1
2010	0.009	85.8	90.4	0.029	89.3	95.1
2020	0.005	86.6	90.4	0.015	83.8	93.3
2030	0.004	87.7	90.4	0.008	76.5	92.3
2040	0.003	91.5	94.9	0.005	67.2	91.3