Solution and Simulation of Large Stock Flow Consistent Monetary Production Models via the Gauss Seidel Algorithm

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1. Introduction

The aim of this paper is to produce a benchmark stock flow consistent model of the type introduced by Godley and Lavoie, (2007), along with an algorithm for the efficient solution of these models.

A stock-flow consistent model is a macrodynamic model based on national income and product accounts and flow of funds accounts. Godley and Lavoie (1999, 2004, 2007) introduce a promising approach to modelling and analyzing national income and product accounts and flow of funds accounts, emphasizing the dynamic interaction between price formation and functional income distribution. Godley and Lavoie's models are set up in such a way as to facilitate the construction of behavioural macroeconomic models in the tradition of Keynes and Kalecki. This model building results in aggregate dynamics that can be described as business cycles, that is, the models replicate observed empirical regularities when parameterised.

Simulation of SFC Models

The extended accounting scheme underpinning any aggregate dynamics simulated in a stock flow consistent model summarizes the transactions taking place within a realistic and modern financial sector that contains securities and financial facilities.

Each model divides the economy into five sectors: firms, households, government, the central bank, and private banks. These sectors are designed in such a way that the income inflows to one sector are outflows from other sectors. The same is true for financial transactions. These flows, when they accumulate over time, as they must in a dynamic context, become stocks; thus, the models are stock flow consistent.

Godley (2005) explicitly recognises the need to outline a theoretical model of a fully established economy, and his 2005 paper concluded that, while the stock flow consistent model presented was another step forward, it still was no more than a sketch.

Zezza and Dos Santos (2008) produce a stock flow consistent model that includes a financial system. The model for that paper was developed from Godley and Lavoie's previous work, and included households, firms, government and banks along with a central bank. Their aim was to further the process in achieving a post-Keynesian growth model that that is both rigorous and flexible enough to be applied to the analysis of macroeconomic policies in actual economies (Salvatore 2002, 2004). They proposed that their model was a starting point for integrating real and financial markets in a post-Keynesian model of a growing economy, but again, they argued much more research was needed to build on that model (Zezza and Dos Santos, 2008).

This paper presents a stock flow consistent growth model of an economy with well-developed financial markets along with many other important characteristics. Production, consumption, and output decisions by sector are illustrated together with increasing productivity growth and mark-up pricing. Building on the need for the integration of a financial system, the models include a through financial system comprised of loans, deposits and credit money. Non-performing loans (*npl*'s) are accounted for throughout the model, which is of relevance to the present economic situation. It can be understood that when firms default on a proportion of their loans, they lose an equivalent value of their inventories to bank confiscation or otherwise. Due to the risk of default, banks need to accurately forecast the value of defaulted loans. By doing this additional profits can, and must, be retained to compensate for the possibility of shortfall allowing own fund targets to be met. In accordance with real world economics banks need to maintain minimum capital requirements set down by the banking regulator. The importance of retained earnings is emphasized throughout the model.

Commercial banks play a central role in the macroeconomic process because they are the link between the aspirations, expectations and actions of the four different sectors. A central bank is included in this model along with a complete set of equations of the governing decisions that central banks have to make (Godley and Lavoie 2007).

Firms have a distinct set of objectives, for example, to make enough profits to pay for growth-maximizing investment. As production and investment take place before any profits can be made there is a systemic need for firms to obtain loans from outside the production sector, which generate credit money endogenously, so there must exist a banking sector. It is important to note that without a fully functioning banking sector, a stock flow consistent model has little or no relevance to modern industrial economies.

In a large stock flow consistent model there is active control by the government through fiscal policy to maintain growth and stability with full employment. Effects of fiscal policies by the government can be investigated in the model by examining outcomes of changes in taxation, government expenditure, and new issues of bills. We describe the model fully in Section 2.

Section 3 describes the simulation details and the Gauss-Seidel procedure for solving large models of this type. We use the solution mechanism to produce a benchmark simulation, and show some simulation results in the concluding Section 4.

1. Model Economy

The main purpose of this paper is to present a model that is as comprehensive as possible based on the literature available. The model can be described as a full post-Keynesian stock flow consistent model that is, while based on previous works, another step towards achieving a full stock flow consistent model that both through and adjustable enough to be applied to the analysis of macroeconomic policies in actual economies. This has been and still is the main aim of such works in the area of stock flow consistent modelling.

The model economy will achieve a steady state once the inflation rate stabilises. This inflation rate stabilises once the exogenous variables like the tax rate are not over exacerbated by fluctuations from other variables. The steady state will result in the economy growing at a fixed or steady rate for as long as the model continues and is not interfered with by exogenous inputs.

1.1. Model description

1.1.1 Households: Income and consumption

The important equations regarding the household section are outlined in this section. Consumption is a major stimulus on an economy and is described in equation 2 as being influenced by expected disposable income, net lending by banks and their accumulated wealth. Real consumption c_k is influenced by propensity to consume out of income α_1 and out of wealth α_2. Price adjusting this gives nominal consumption *cons*. Expected real regular yd_{kre} is a weighted average of the real regular disposable income of the current and previous period weighted for productivity growth. The nominal wealth of households given by equation 3 is previous wealth, disposable income, capital gains and change in own bank funds minus consumption spending. The demand for cash is assumed not to be related to wealth but related to the flow of consumption spending i.e. consumers only need cash for a proportion of what they consume.

$$yd_{k_{r_e}} = \varepsilon * yd_{k_r} + (1-\varepsilon) * yd_{k_{r-1}} * (1 + gr_{pr}) \tag{1}$$

$$c_k = \alpha_1 * \left(yd_{k_{r_e}} + nl_k\right) + \alpha_2 * v_{k-1} \tag{2}$$

$$v = v_{-1} + yd_r - cons + \Delta p_e * e_{k_{d-1}} + \Delta p_{bl} * bl_{s-1} + \Delta of\_b \tag{3}$$

Nominal personal income yp of households is the sum of wages from firms plus dividends and interest payments on received on deposits, bills and bonds. This personal income is taxed by the government at the taxation rate θ.

Nominal regular disposable income yd_r is personal income minus taxes minus interest payments on loans and is shown in equation 5. Real regular disposable income yd_{k_r} is disposable income minus the capital loss effects of price inflation.

$$yp = wb + fd_f + fd_b + r_{m_{-1}} * m_{d_{-1}} + r_{b_{-1}} * b_{hd_{-1}} + bl_{s_{-1}} \tag{4}$$

$$yd_r = yp - t - r_{l_{-1}} * l\_hd_{-1} \tag{5}$$

$$yd_{k_r} = {}^{yd\_r}/_{p} - \pi * v_{k_{-1}}/p \tag{6}$$

1.1.2 Households: Personal loan decisions

The following equations give a summary of decisions relating to personal financial management. Equation 7 states that the gross amount of personal loans is a proportion η of regular disposable income. The fraction η in equation 8 is governed by the real interest rate on loans.

$$gl = \eta \times yd_r \quad (7)$$

$$\eta = \eta_0 - \eta_r * rr_l \quad (8)$$

The quantities of personal loans that are repaid are determined by an exogenously set proportion of loans held by households in the previous period. The net amount of new personal loans is gross amount of personal loans minus those repaid in the current year which can be price adjusted to give the real amount of personal loans nl_r. Thus the resulting demand for personal loans l_{hd} is previous demand plus net amount of new personal loans illustrated in equation 11.

$$rep = \Delta rep \times l_{hd\,-1} \quad (9)$$

$$nl = gl - rep \quad (10)$$

$$l_{hd} = l_{hd\,-1} + gl - rep \quad (11)$$

The burden of personal debt *bur* on households given by equation 12 is the sum of interest payments and principal repayments as a fraction of disposable income. Essentially it is the burden of servicing the debt with respect to real disposable income.

$$bur = \frac{rep + r_{l\,-1} \cdot l_{hd\,-1}}{yd_{r\,-1}} \quad (12)$$

1.1.3 Households: Investment and capital gains

The financial markets in model economy are made up of four kinds of assets: money deposits, bills, bonds and equities. Households make portfolio decisions regarding the allocation of their expected wealth between the various assets available to them. The investible wealth of households is their wealth plus liabilities minus wealth that is in the form of cash minus wealth that is needed to capitalise the banks.

$$v_{fma} = v + l_{hd} - h_{hd} - of_b \tag{13}$$

From this investible wealth v_{fma} households can decide what assets to invest in. The demand for the four different assets, money deposits m_d, bills b_{hd}, bonds b_{ld} and equities p_e in a given period is given in equations 14 - 17. The choice on asset depends on the real expected return on that asset. Wealth is allocated between assets on Tobinesque principles and the equations for this are taken directly from Godley and Lavoie (2007).

$$m_d = v_{fma} - b_{hd} - p_e * e_{k_d} - p_{bl} * bl_d + l_{hd} \tag{14}$$

$$b_{hd} = v_{fma_{-1}} * \left(\lambda_{20} + \lambda_{22} * r_{b_{-1}} - \lambda_{21} * r_{m_{-1}} - \lambda_{24} * r_{k_{-1}} - \lambda_{23} * r_{bl_{-1}} - \lambda_{25} \frac{yd_r}{v_{fma_{-1}}}\right) \tag{15}$$

$$bl_d = \frac{v_{fma_{-1}} * (\lambda_{30} - \lambda_{32} * r_{b_{-1}} - \lambda_{31} * r_{m_{-1}} - \lambda_{34} * r_{k_{-1}} + \lambda_{33} * r_{bl_{-1}} - \lambda_{35} \frac{yd_r}{v_{fma_{-1}}})}{p_{bl}} \tag{16}$$

$$p_e = \frac{v_{fma_{-1}} * \left((\lambda_{40} - \lambda_{42} * r_{b_{-1}} - \lambda_{41} * r_{m_{-1}} + \lambda_{44} * r_{k_{-1}} - \lambda_{43} * r_{bl_{-1}}) - \lambda_{45} * \left(\frac{yd_r}{v_{fma_{-1}}}\right)\right)}{e_{kd}} \tag{17}$$

The resulting capital gains of households is given by the capital gains on bonds and equities plus the change in the own funds of banks which belongs to households. Haig-Simons measure of nominal disposable income is regular disposable income plus these capital gains

$$cg = \Delta p_{bl} * bl_{d_{-1}} + \Delta p_e * e_{k_{d_{-1}}} + \Delta of_b \tag{18}$$

$$yd_{hs} = yd_r + cg \tag{19}$$

1.2.1 Firms

In model economy firms make decisions regarding how much they are going to produce, inventory they are going to keep, quantity of people to employ, how much investment and where the funding for this will come from e.g. retained profits or issue of shares.

1.2.2 Firms: Output and inventories

The real output y_k by firms is given by equation 20.

It states that output is the sum of expected sales plus the expected change in inventories. Expected real sales s_{ke} is a function of real sales and the growth of productivity

Actual real sales s_k are a summation of consumption, government expenditure and gross investment as shown by equation 22.

$$y_k = s_{k_e} + in_{k_e} - in_{k_{-1}} \tag{20}$$

$$s_{k_e} = \beta * s_k + (1 - \beta) * s_{k_{-1}} * (1 + gr_{pr}) \tag{21}$$

$$s_k = c_k + g_k + i_k \tag{22}$$

Firms manage their inventory levels using the following equations 23 and 24. The inventories are a safeguard of unexpected changes in demand and so act as a buffer. The long run inventory target in_{kt} that The long run inventory target in_{kt} that firms seek to have is an exogenously set proportion σ^T of expected real sales which in model economy is set at $\sigma^T = 0.2$. The short term inventory target in_{ke} which firms seek to have at the end of the current year is determined by applying an exogenous adjustment factor $\gamma = 0.15$ towards achieving the long run inventory target.

$$in_{k\_t} = \sigma^{T} * s_{k_e} \tag{23}$$

$$in_{k_e} = in_{k_{-1}} + \gamma * (in_{k_t} - in_{k_{-1}}) \tag{24}$$

The actual real inventories are the sum of previous inventory plus current output minus current sales. The formula also takes into account for the effect on inventories of non-performing loans by subtracting the amount of non-performing loans in inventory terms. If firms have €*npl* amount of non-performing loans this means they should lose *npl/uc* units of inventories either to the banks or because the merchandise is out-dated. Multiplying the quantity of inventories by the unit cost yields the total value of inventories held illustrated by equation 26.

$$in_k = in_{k_{-1}} + y_k - s_k - {}^{npl}/_{uc} \tag{25}$$

$$in = in_k * uc \tag{26}$$

Nominal GDP is one of the most important measures of an economy and is given by equation 27 as the present value of real sales and the value of additional inventory stock. The capital utilisation proxy *u* is an important measure of a firms performance and is illustrated in equation 28 as the amount by which firms make use of their real capital stock to produce real output.

$$y = s_k * p + uc * (\Delta in_k) \tag{27}$$

$$u = {}^{y_k}/_{k_{k_{-1}}} \tag{28}$$

1.2.3 Firms: Pricing and unit costs

Firms also must make pricing decisions in order to achieve sales to pay for all expenditures and also generate enough profit to satisfy shareholders and pay for some considerable proportion of investment. Firms must make these decisions on their forecasting of expected sales, s_{ke}. In order to grow, firms must raise their prices every year. Equation 29 illustrates that prices p is based on normal historical unit costs, to which a mark-up φ is applied. The actual price mark-up φ is determined by equation 30 which factors in the ideal mark-up rate, previous mark up and an adjustment factor ($\varepsilon_2 = 0.8$). The ideal price mark-up φ_t is the ratio of targeted entrepreneurial profits over expected historic costs. The idea of the target mark-up is to meet the target amount of entrepreneurial profits which real sales equals expected sales, thus expected historic unit costs would be equal to historic unit costs. This price increase results in the inflation rate π defined of the rate of change in prices p.

$$p = (1 + \varphi) * nhuc \tag{29}$$

$$\varphi = \varphi_{-1} + \varepsilon_2 * (\varphi^T_{-1} - \varphi_{-1}) \tag{30}$$

$$\varphi_t \quad {}^{fr_t}\!/_{hc_e} \tag{31}$$

$$\pi = {}^{\Delta p}\!/_{p_{-1}} \tag{32}$$

The actual unit cost *uc* is the wage bill divided by the output while the normal unit cost *nuc* is simply wage rate over productivity. The difference is that *nuc* determines normal historic unit cost which is based on past and current normal costs, what firms believe to be normal inventories to sales target σ^N and the normal interest rate r_{ln}.

$$uc = {}^{wb}\!/_{y_k} \tag{33}$$

$$nuc = {}^{W}/_{pr} \tag{34}$$

$$nhuc = nuc * (1 - \sigma^N) + nuc_{-1} * \sigma^N * (1 + r_{ln}) \tag{35}$$

The expected historic unit cost used in the determination of the ideal mark-up is given by equation 36. While the opening inventories to expected sales ratio is given by equation 37.

$$hc_e = uc * s_{k_e} * (1 - \sigma_{se}) + \sigma_{se} * \left(1 + r_{l_{-1}}\right) * s_{k_e} * uc_{-1} \tag{36}$$

$$\sigma_{se} = {}^{in_{k_{-1}}}/_{s_{k_e}} \tag{37}$$

1.2.4 Firms: Wages and employment

Workers real wage aspirations are found using the logarithmic equation 38 (Zezza 2006). The value for ω_t depends on trend labour productivity and the demand for labour. The equation also incorporates the Philips curve which states that when the employment rate is within a certain band there are no additional pressures on the target real wage rate. The nominal wage is found by taking the difference between a price adjusted target wage and previous nominal wage, multiplying this by a speed of adjustment ω_3, and adding the result to the previous period's nominal wage.

$$\omega_t = \exp\left(\omega_0 + \omega_1 \log(pr) + \omega_2 \log\left(er + z_3(1 - er) - z_4 bandT + z_5 bandB\right)\right)$$

(38)

$$W = W_{-1} + \omega_3 * (\omega_t * p_{-1} - W_{-1}) \tag{39}$$

The desired employment level is determined by equation 40 and is directly influenced by the growth of labour productivity while the actual employment level is found using equation 41. The wage bill given by equation 42 is understandably the number of workers times the nominal wage.

$$n_t = {}^{y_k}/_{pr} \tag{40}$$

$$n = n_{-1} + \eta_n * (n_t - n_{-1}) \tag{41}$$

$$wb = n * w \tag{42}$$

1.2.5 Firms: Investment and profits

The growth of real capital stock gr_k depends on the real rate of interest rr_l and the capital utilisation u.The real capital stock k_k grows by the variable rate gr_k per annum which can be price level adjusted to give nominal value of fixed capital K. Real gross investment i_k grows at a rate gr_k plus rate of accumulation of capital δ every year. This can be price level adjusted to give nominal gross investment I.

$$gr_k = \gamma_0 + \gamma_u * u_{-1} - \gamma_r * rr_l \tag{43}$$

$$k_k = k_{k_{-1}} * (1 + gr_k) \tag{44}$$

$$i_k = (gr_k + \delta) * k_{k_{-1}} \tag{45}$$

The planned retained earnings of firms fu_{f_t} are a proportion φ_u of gross investment for the previous. While equation 47 illustrates that the dividend paid by firms is a proportion φ_d of the previous year's profits.

$$fu_{f_t} = \psi_u * I_{-1} \tag{46}$$

$$fd_f = \psi_d * f_{f_{-1}} \tag{47}$$

The targeted entrepreneurial profits f_{f_t} is so as to cover the sum of dividends paid out, expected profits of firms and the interest payment on loans other than those generated by inventories. The actual Realised entrepreneurial profits f_f is sales minus wage bill plus additional inventory minus the financing cost of holding previous inventories

$$f_{f_t} = fu_{f_t} + fd_f + r_{l_{-1}} * (l_{fd_{-1}} - in_{-1}) \tag{48}$$

$$f_f = s_k * p - wb + \Delta in - r_{l_{-1}} * in_{-1} \tag{49}$$

Retained earnings of firms is given by the entrepreneurial profits minus the dividends paid out minus the interest payments on loans other than those generated by inventories, plus an additional term r_{l-1}*npl which is the loan repayments that firms defaulted on. This additional term is because since the firms have defaulted on €npl of loans they do not pay interest on them thus retain € r_{l-1}*npl.

$$fu_f = f_f - fd_f - r_{l_{-1}} * \left(l_{fd_{-1}} - in_{-1}\right) + r_{l_{-1}} * npl \tag{50}$$

Firms require loans for the creation of inventories and also if funding from share issues and retained profits did not meet targets. Demand for loans by firms is previous demand plus gross investment and increases in inventory levels minus profits from the issue of shares and retained earnings. Firms also decrease the need for more funds when they default on loans. Using fisher's equation for determining the real interest rate yields equation 53. This formula real interest rate rr_l on loans is a function of the interest rate and the level of price inflation

$$l_{fd} = l_{fd_{-1}} + I + \Delta in - fu_f - \Delta e_{k_s} * p_e - npl \tag{51}$$

$$rr_l = {}^{1 + r_l}/_{1 + \pi} - 1 \tag{52}$$

The dividend yield of firms is given by equation 53 and is the ratio of the dividends paid by firms to the overall market value of shares outstanding at the end of the previous period. The price earnings ratio given by equation 54 is the current equity price divided by profits per share. Tobin's q ratio is given by equation 55 as the ratio between the market value of outstanding equities and replacement value of real capital stock, inventories and loans demanded.

$$r_k = \frac{fd_f}{p_{e_{-1}} * e_{kd_{-1}}} \tag{53}$$

$$pe = {p_e}\Big/\frac{f_f}{ek_{s-1}} \qquad (54)$$

$$q = \frac{ek_s \cdot p_e}{k + in + l_{fd}} \qquad (55)$$

1.3.1 Government

The government is assumed to be able to determine its own real expenditure g_k on goods and services. Real government expenditures g_k grow per annum by a rate of gr_g which is set exogenously in the model at 3%. This can be price adjusted to give pure government expenditures g.

$$g_k = g_{k_{-1}} * (1 + gr_g) \qquad (56)$$

The government deficit $psbr$ is the sum of government expenditure, supply of bonds and the interest payments on bills supplied to households and banks minus the taxes they take in every fiscal year. The taxes rate Θ is set exogenously in the model. Government debt gd is the net sum of bills supplied to banks and households, the total value of bonds outstanding and the supply of money by the central bank.

$$psbr = g + bl_{s_{-1}} + r_{b_{-1}} * \left(b_{bs_{-1}} + b_{hs_{-1}}\right) - t \qquad (57)$$

$$gd = b_{bs} + b_{hs} + bl_s * p_{bl} + h_s \qquad (58)$$

The new issue of bills of bills is the net amount of previous bills and bonds plus government expenditure and interest payments on bills supplied to households and banks minus taxes and the value of new bonds.

$$b_s = b_{s_{-1}} + g - t - p_{bl} * \Delta bl_s + r_{b_{-1}} * \left(b_{bs_{-1}} + b_{hs_{-1}}\right) + bl_{s_{-1}} \qquad (59)$$

1.4.1 Commercial banks

Banks play a critical role in the modern industrial economy and model economy. Banks are subject to a minimum capital requirement set down by the national or international international regulator so they are always trying to meet this target. The long run own funds target of_{b_T} as described by equation 60 is the amount of capital banks are required to hold which is a proportion *ncar* of the loans supplied to households and firms. The banks do not immediately respond to the long term capital requirements so the short run own funds target of banks of_{b_S} is the amount by which banks aim to meet their long term funds target which is $\beta_b = 0.4$. The own funds of banks at the end of the year are the previous own funds plus retained earnings minus the net amount of defaulted loans.

$$of_{b_T} = ncar * (l_{fs_{-1}} + l_{hs_{-1}}) \tag{60}$$

$$of_{b_S} = of_{b_{-1}} + \beta_b * (of_{b_T} - of_{b_{-1}}) \tag{61}$$

$$of_b = of_{b_{-1}} + fu_b - npl \tag{62}$$

In order to make profits banks must set interest rates in a way that ensures that they make profits the sum of their interest receipts must exceed the sum of interest payments by a margin that is acceptable to shareholders. They also must ensure that enough profits are generated to cover short-term capital requirements. The target profits banks aim to achieve are so as to cover dividend payments; short term own funds targets and defaulted loans. Equation 64 illustrates the rate of interest that banks pay on deposits. The model adjusts the variables so as to keep the bank liquidity ratio approximately 0.05 to 0.12. . Equation 65 gives the lending mark up over deposit rate add_l which will allow banks to meet their profit and capital adequacy objectives. This is then added to the deposit interest rate to give the loan interest rate in equation 66 thus the difference resulting in a profit for the banks.

$$f_{b_t} = f_{u_b} + \mathrm{of}_{b_e} - \mathrm{of}_{b_{-1}} + \mathrm{npl}_{ke} * \mathrm{l}_{fs_{-1}} \quad (63)$$

$$r_m = r_{m_{-1}} + z_{1_a} * xim_1 + z_{1_b} * xim_2 - z_{2_a} * xim_1 - z_{2_b} * xim_2 \quad (64)$$

$$add_l = \frac{f_{b_t} - r_{b_{-1}} \cdot b_{bd_{-1}} + r_m \cdot (m_{s_{-1}} - (1 - npl_{ke}) \cdot l_{fs_{-1}} - l_{hs_{-1}})}{(1 - npl_{ke}) \cdot l_{fs_{-1}} - l_{hs_{-1}}} \quad (65)$$

$$r_l = r_m + add_l \quad (66)$$

The bank liquidity ratio gives the bill to deposit ratio which is used to determine deposit interest rate in order to keep the liquidity ratio within an acceptable range which in model economy is set as 0.05 – 0.12. At the end of a given period equation 67 determines the actual capital capacity ratio, which can be compared with the desired proportion *ncar*.

$$blr = \frac{b_{bd}}{m_s} \quad (67)$$

$$car = \frac{of_b}{l_{fs} + l_{hs}} \quad (68)$$

To ensure profitability banks must facilitate for the risk of defaulted loans. The total amount of defaulted loans *npl* is a fraction npl_k of the loans supplied to firms. The expected proportion of non-performing loans is given by equation 69. It is determined by a weighting $\varepsilon_b = 0.25$ which weights the result between previous estimates and actual values for defaulted loans in previous periods.

$$npl = npl_k * l_{fs_{-1}} \quad (69)$$

$$npl_{ke} = \varepsilon_b * npl_{ke_{-1}} + npl_{k_{-1}}(1 - \varepsilon_b) \quad (70)$$

The targeted retained earnings of banks is the amount needed to mean short term own funds targets plus the amount needed to cover the amount of own funds lost through bad loans. The actual profits of banks are obtained from loan repayments from households and firms that have not defaulted plus the revenue generated from bills they hold minus interest paid on deposits. The actual retained earnings of banks are the actual profits minus any dividends paid out.

$$fu_{b_t} = of_{b_e} - of_{b_{-1}} + npl_{ke} + l_{fs_{-1}} \tag{71}$$

$$f_b = r_{l_{-1}} * \left(l_{fs_{-1}} + l_{ks_{-1}} - npl\right) + r_{b_{-1}} * b_{bd_{-1}} - r_{m_{-1}} * m_{s_{-1}} \tag{72}$$

$$fu_b = f_b - fd_b$$

(73)

The dividend paid out by banks is a predetermined proportion λ_b of output y of the previous period.

$$fd_b = \lambda_b y_{-1} \tag{74}$$

The bills supplied to banks b_{bs} are the bills needed to supply households, the central bank and the bank themselves as illustrated in equation 75. The balance sheet constraint of banks b_{bd} in equation 76 is the amount of bills it needs to satisfy reserve requirements, own funds requirements and money holdings while fulfilling loans demands.

$$b_{bs} = b_{bs_{-1}} + \Delta b_s + \Delta b_{hs} + \Delta b_{cbs} \tag{75}$$

$$b_{bd} = m_s + of_b - l_{fs} - l_{ks} - h_{bd} \tag{76}$$

1.5.1 Central bank

Central banks have a simple role in model economy as it only holds treasury bills as assets while its liabilities are only made up of bank reserves and bank notes. Central bank profits are the interest received on government bills as given by equation 77. The model economy has a number of supply equals demand assumptions with regard to bonds, household bills and cash while it buys the bills that it demands

$$f_{cb} = r_{b_{-1}} * b_{cbd_{-1}} \tag{77}$$

The long term interest rate on bonds r_{bl} is assumed to be the interest rate on bills plus a fixed mark-up. The interest rate on bills is set exogenously. The price of long term bonds p_{bl} is the reciprocal of the long term interest rate.

$$r_{bl} = r_b + add_{bl} \tag{78}$$

$$p_{bl} = {}^{1}/_{r_{bl}} \tag{79}$$

This closes the model.

2. Model solution

The model economy was computed in Matlab 7.10.0 which is a high-level language and interactive environment that enables computation of mathematically intensive tasks. Matlab models algorithms sequentially so this had to be taken into account when inputting the model economy equations. The Matlab code for this model is available from the authors.

An economic year is completed by going through model economy once. The above model equations in section 2.1 are within a for loop that repeats the equations j times with each value of j being another year. The advantage of this is that exogenous variables such as the taxation rate θ can be easily changed for each loop i.e. mid simulation thus making more complex experiments possible. The entire simulation is placed within two-dimensional matrices that increase in size with each value of j.

For example $er(j)$ stores the value of the employment rate for the j^{th} loop in the 1xj space in the er matrix.

The problem with the above economic model however is that it cannot be easily solved because many of the equations are interdependent. Much of the model was rearranged sequentially but this did not solve the issue of the interdependent equations. A recursive algorithm is necessary for solving the linear system of equations.

The Gauss Seidel method is an iterative algorithm used to solve a linear system of equations. It works by using an initial guess to solve a system of equations and iterating these equations until a satisfactory convergence is reached. Convergence is only guaranteed if the matrix is either diagonally dominant, or symmetric and positive definite.

Linear systems of equations take the general form $Ax = b$, where A is $n \times n$ and non-singular. The i^{th} equation can be written as

$$\sum_{j=1}^{n} a_{i,j} x_j = b_i$$

The i^{th} equation is used to modify the i^{th} unknown with the iteration completed sequentially from $i = 1, \ldots, n$. The first equation is used to compute x_1 , the second equation to compute x_2 and so on. In computing the i^{th} equation variables $x_1, x_2, \ldots, x_{i-1}$ have already been computed. These are used in the calculation of x_i rather than the previous iteration values (Watkins, D.S 2002).

2.1 A simple example

The following is a simple example set of equations that can be solved by the Gauss-Seidel algorithm.

$$\begin{bmatrix} 10 & -1 & 2 & 0 \\ -1 & 11 & -1 & 3 \\ 2 & -1 & 10 & -1 \\ 0 & 3 & -1 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 6 \\ 25 \\ -11 \\ 15 \end{bmatrix}$$

It can be shown that the above set of equations are diagonally dominant and so can be solved using the Gauss-Seidel algorithm. Each equation is then written in terms of x_1, x_2, x_3 and x_4.

$$x_1 = \frac{6 - x_2 + 2x_3}{10}$$

$$x_2 = \frac{x_1 + x_3 - 3x_4 + 25}{11}$$

$$x_3 = \frac{-2x_1 + x_2 + x_4 - 11}{10}$$

$$x_4 = \frac{15 - 3x_2 + x_3}{8}$$

If the physics of the problem are understood, a close initial guess can be made, decreasing the number of iterations needed. In this case the initial guess will be set to,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Substituting this initial guess into the previous equations yields a first iteration result of;

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0.6 \\ 2.27 \\ -1.1 \\ 1.87 \end{bmatrix}$$

Using these values a second iteration can be done yielding;

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1.04 \\ 1.71 \\ -0.8 \\ 0.88 \end{bmatrix}$$

This process is repeated until satisfactory convergence is reached. The iterations are stopped when the absolute relative approximate error is less than a pre-specified tolerance for all unknowns.

$$error < tolerence$$

$$error_i = \left| \frac{x_i - x_{i-1}}{x_i} \right| \times 100$$

Continuing the above iteration process yields a solution that converges towards;

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 1 \end{bmatrix}$$

Code 1 illustrates the Matlab algorithm that solves the above equations using the Gauss are used within each iteration. The loop is specified to iterate the equations 100 times but *1* reaches a convergence tolerance of 0.1% the iteration process ceases.

```
x1=1;
x2=1;
x3=1;
x4=1;
for z=2:100

            x1(z) = x2(z-1)/10 - x3(z-1)/5 + 3/5
        x2(z) = x1(z)/11 + x3(z-1)/11 - (3*x4(z-1))/11 +25/11
        x3(z) = -x1(z)/5 + x2(z)/10 + x4(z-1)/10 - 11/10
        x4(z) = -(3*x2(z))/8 + x3(z)/8 + 15/8

if abs(x1(z)-x1(z-1))/x1(z)<0.001
```

```
break
end
end
```

Code 1 Matlab code for solution of sample Gauss-Seidel algorithm

In order to simulate multiple periods/years and to carry out the Gauss-Seidel algorithm it is clear that 2 loops are need. The Gauss Seidel algorithm was applied to model economy in order to solve it by placing the model equations within a second z loop. The z loop acts as an iterative loop within the j loop which allows for the Gauss-Seidel algorithm to be applied. Code 2 clarifies how the model equations were placed in the double loop. Every time the j loop repeats the model is advanced a year before the iteration begins.

```
for j=2:100
year(j)=year+j

        for z=2:20
                Model economy equations
        end
end
```

Code 2 Model equations placed within double loop

When the model inters its first iteration initial guesses are made for six variables c_k, p_e, e_{kd}, r_l, r_m. The most accurate initial guess is to set the variables equal to the values from the previous period. This initial guess significantly reduces the amount of iterations needed. For the purpose of the iteration the variables names were changed by adding a z so as not to interfere with the j loop matrixes.

```
c_kz(z-1)=c_k(j-1);
e_k_dz(z-1)=e_k_d(j-1);
p_ez(z-1)=p_e(j-1);
r_lz(z-1)=r_l(j-1);
r_mz (z-1) =r_m (j-1);
```

Code 3 Initial guesses

When the algorithm is processing the equations it uses the previous values for the iterated variables in the current iteration i.e. c_kz(z-1). This is why the initial guesses need to be entered as if it's a previous iteration e.g. c_kz(z-1)=c_k(j-1);. Also for this reason the z loop needs to begin at z=2. If the z loop began at z=1 the model would attempt to input the initial guess into the zero element of a matrix which would result in an error.

The z loop repeats the model equations until the values for these variables converge to give the new values for the current period. The accuracy of convergence is checked by monitoring each iteration process. If the set tolerance of $tol = 0.01\%$ is reached the z loop is terminated thus ending the iteration process for that j loop/year. Code 4 displays the convergence examination procedure.

```
tolc_kz=abs(c_kz(z)-c_kz(z-1))/c_kz(z);
tole_k_dz=abs(e_k_dz(z)-e_k_dz(z-1))/e_k_dz(z);
tolp_ez=abs(p_ez(z)-p_ez(z-1))/p_ez(z);
tolr_lz=abs(r_lz(z)-r_lz(z-1))/r_lz(z);
tolr_mz=abs(r_mz(z)-r_mz(z-1))/r_mz(z);

if tolc_kz<tol&& tole_k_dz<tol&& tolp_ez<tol&& tolr_lz<tol&& tolr_mz <tol
   break
end
```

Code 4 Tolerance is checked for each variable

Once the iteration process converges to the satisfactory tolerance the values for the iterated variables are then assigned to their respective j loop matrixes. The iterated variables are then deleted to allow for the next iteration process.

```
c_k(j)=c_kz(z);
e_k_d(j)=e_k_dz(z);
p_e(j)=p_ez(z);
r_l(j)=r_lz(z);
r_m(j)=r_mz(z);
clear c_kz e_k_dz p_ez r_lz r_mz
```

Code 5 Iterated values stored once convergence is reached

The model then enters another j loop /year and the iteration process will repeat itself again. Once the j loops are completed graphs can then be made of the results.

Code 6 illustrates the full Gauss-Seidel algorithm less the model equations. It can be seen that every j loop accounts for another year. When the model begins its first iteration for a particular year the condition z=2 is met so the initial guesses are made. The iteration process then continues until the required tolerance level is reached. This then activates the break condition, which terminates the iteration process. The values for the iterated variables are then stored and the model then begins another year.

```
for j=2:100
year(j)=year+j

    for z=2:20
        if z==2
        c_kz(z-1)=c_k(j-1);
```

```
                e_k_dz(z-1)=e_k_d(j-1);
                p_ez(z-1)=p_e(j-1);
                r_lz(z-1)=r_l(j-1);
                    r_mz (z-1) =r_m (j-1);
                    end
                --------------------------------------
```

Model economy equations

```
                --------------------------------------

                    tolc_kz=abs(c_kz(z)-c_kz(z-1))/c_kz(z);
                    tole_k_dz=abs(e_k_dz(z)-e_k_dz(z-1))/e_k_dz(z);
                    tolp_ez=abs(p_ez(z)-p_ez(z-1))/p_ez(z);
                    tolr_lz=abs(r_lz(z)-r_lz(z-1))/r_lz(z);
                    tolr_mz=abs(r_mz(z)-r_mz(z-1))/r_mz(z);

if tolc_kz<tol&& tole_k_dz<tol&& tolp_ez<tol&& tolr_lz<tol&& tolr_mz <tol
   break
end

                end

            c_k(j)=c_kz(z);
        e_k_d(j)=e_k_dz(z);
        p_e(j)=p_ez(z);
        r_l(j)=r_lz(z);
        r_m(j)=r_mz(z);
            clear c_kz e_k_dz p_ez r_lz r_mz

            end
```

Code 6 Gauss-Seidel algorithm

3. Results

3.1 Gauss-Seidel algorithm

Figure 1 displays the results of the iteration process when a convergence tolerance of 10% is set. The number of iterations required fluctuates between two and three. This is understandably because of the inaccuracy of solution required. Obviously accuracies of 10% are of little benefit but this is for comparison with figure 2.

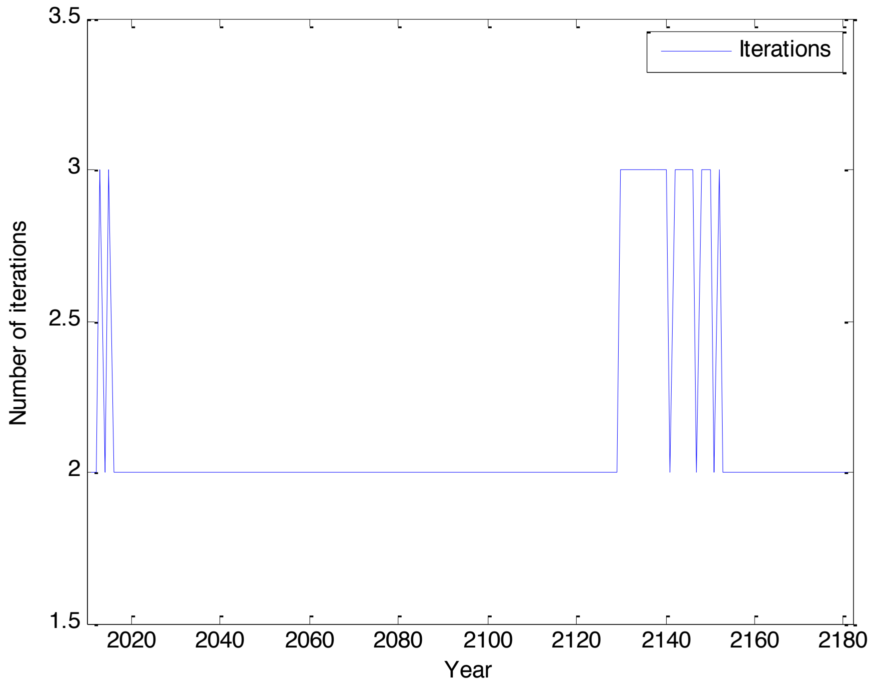

Figure 1 Number of iterations completed for a convergence tolerance level of 10%

The tolerance level was then improved to 0.0001%, the results of which can be seen in figure 2. The number of iterations required oscillates between six and seven signifying the stability and consistency of the algorithm. This illustrates how efficient the Gauss-Seidel algorithm is at achieving accurate results.

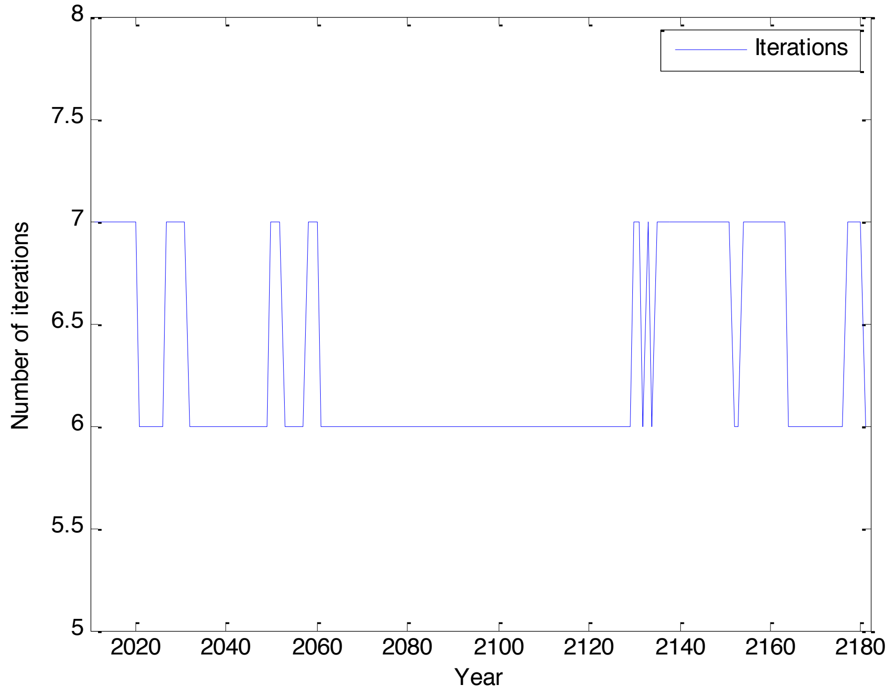

Figure 2 Number of iterations completed for a convergence tolerance level of 0.0001%

3.2 Model economy results

Figure 3 illustrates the main characteristics of the model economy. The model achieves a steady state with 2% inflation, 3% productivity and wage aspiration growth, and 5% wage inflation growth.

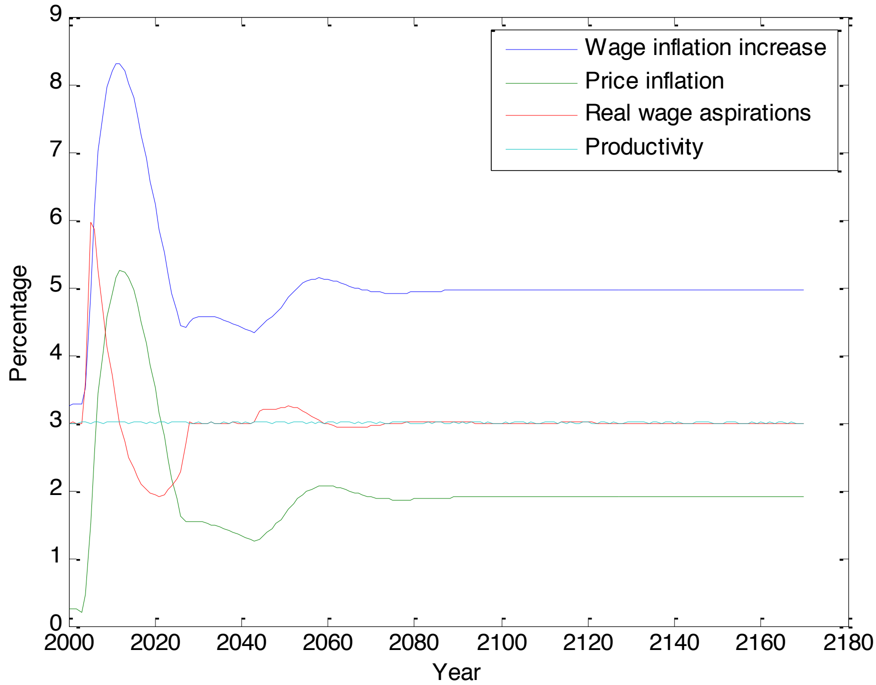

Figure 3 Percentage increases of wage, price inflation, wage aspirations and productivity

Figure 4 illustrates that once the model achieves steady state, real consumption, sales and output converge at 3% growth per annum.

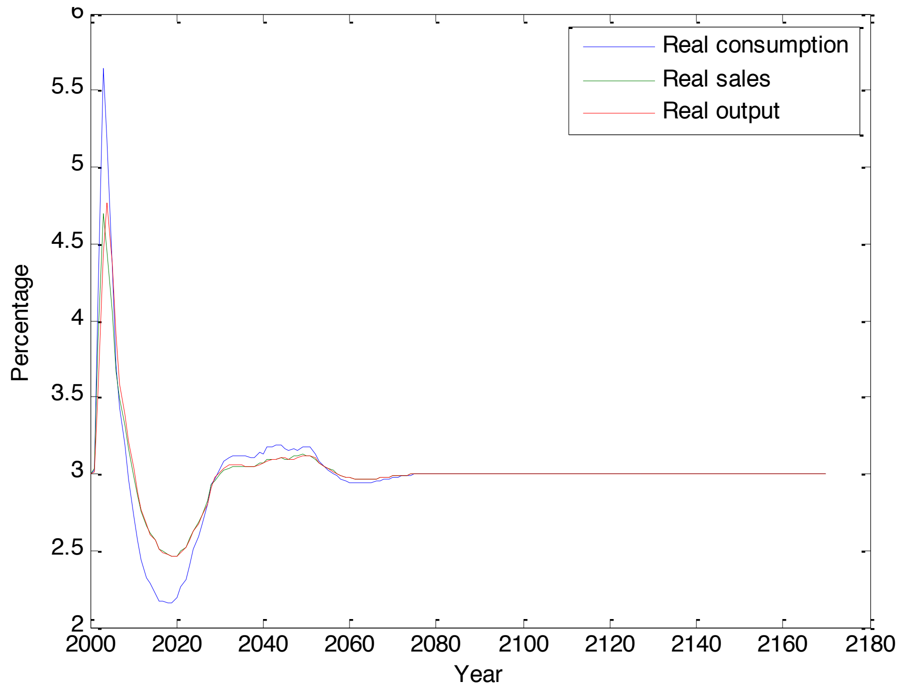

Figure 4 Percentage increases of real consumption, sales and output

Figure 5 shows that full employment is achieved in the model which is one of the aims of Post-Keynesian economics. The fact that the employment rate is slightly greater than full employment means that more people are employed tahn what is considered full employment in the model.

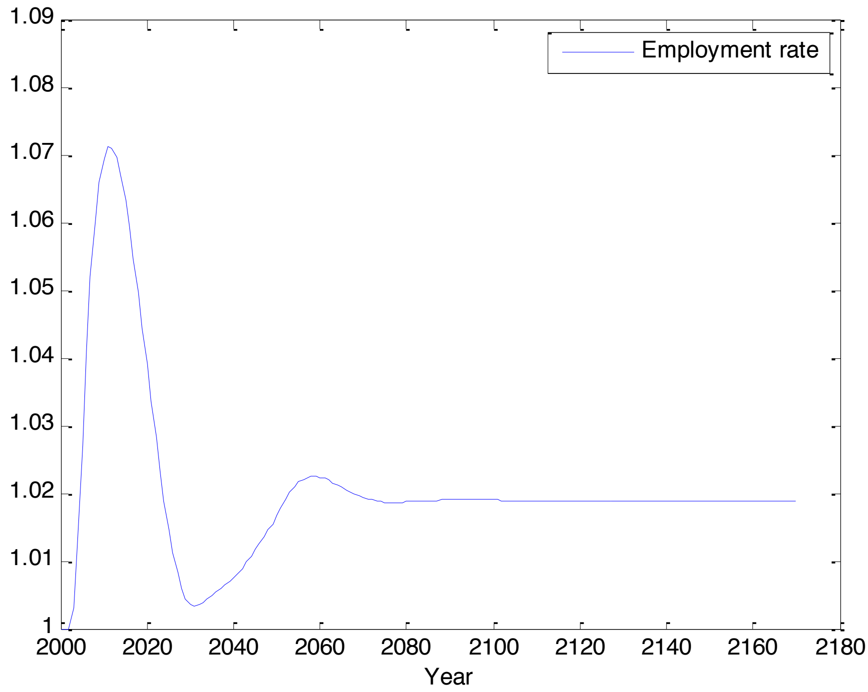

Figure 5 Employment rate

4. Discussion and further work

This paper presents a stock flow consistent model for a closed economy, describing the main relations among households, firms, banks, the government and a central bank. Few stock flow consistent models exist in the literature that are comparable with this model economy. The significance is that many commonly ignored aspects like financial markets and loan defaulting are included. The effects of non-performing loans which is of significant relevance to present economics can be seen throughout the stock flow consistent model.

The paper also presents a technique of overcoming the problem of complexity associated with large stock flow consistent models, which is the Gauss-Seidel algorithm, the use of which paves the way for much more complex stock flow consistent models to be developed. This is a significant property of the stock flow consistent model which allows for expansion by including more complex equations. The analysis of fiscal policies can also be accomplished due to the design of the model. Taxation rates, base interest rates etc. can be adjusted mid simulation or year to year making the model more realistic.

The model is demand led in that it assumes that any demands are met with the necessary supplies of cash, bills, etc. This assumption is used occasionally throughout the model and does not in any way undermine its integrity. The model can also be used to investigate the credibility of other simpler demand led models, allowing for the evaluation of simulations carried out with such models. This could show that results of simulations from simpler stock flow consistent models change when allowing for more complex interactions. The inclusion of loan defaulting and financial markets are examples of such interactions.

This paper is another step towards the creation of a full stock flow consistent Post-Keynesian model of a growing economy, allowing for the integration of financial markets. The importance of such integration cannot be underestimated in modern industry because of the vast quantities of financial securities being traded globally. Economics has dramatically changed in recent times due to the exponential growth of security trading in which vast quantities of wealth are exchanged. It is hoped that this paper will be of benefit to future works in this area that could build on this model.

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