The Internal-External Debt Ratio and Economic Growth

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Abstract

In this paper we examine the effects of the ratio of internal to external public debt on a country's economic growth. These effects are examined through a competitive, decentralized model of endogenous economic growth, which relies on public investments. Our findings show that as the internal-external public debt ratio increases, the public to private capital ratio increases which in turn positively affects the long run economic growth rate. The main conclusion of this paper is that the outflow of domestic capital which is needed to service external debt has unfavorable repercussions on an economy's long run steady state growth rate.

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1. Introduction

It is a well-known fact that many developing countries are faced with high levels of indebtedness (relative to their GDP). This issue has gained much at-

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tention during the recent decades as many countries have experienced lasting budget deficits which lead to sharp increases in debt-to-GDP ratios, and for many, a large share of external debt.[1] Many authors argue that these high levels of debt (especially external) have a negative effect on economic growth (Afxentiou, 1993). This negative relationship is often attributed to "'debt overhang"' which is defined as the situation in which the expected repayment on external debt falls short of the contractual value of debt, and therefore expected debt service is likely to be an increasing function of the country's output level (Krugman, 1988). These high debt servicing costs place a strain on the fiscal situation of the country and on the overall prospects of its economy.

Some authors claim that this relationship occurs as high levels of debt lead to a reduction in private investment, thus the lower growth rates. Savvides (1992), IMF (1989) and Greene & Villanueva (1991) find that debt overhang is a significant factor influencing slowdown in private investment. Nguyen et al. (2004) show that although the stock of public debt does not appear to depress public investment, the level of debt service does. In this context, debt is considered to imply a future tax on output which leads to a discouragement of private sector investment plans and adjustment efforts on the part of governments. Furthermore, Agénor & Montiel (1996) argue that is is more likely that these debt servicing obligations will be financed

[1] In the early 1980s many Latin American countries were engulfed in a financial crises when international capital markets became aware that Latin America would probably not be able to pay back its loans. This lead to economic growth stagnation and increased unemployment in those countries (amongst other dire effects) for at least a decade.

though distortionary measures (e.g. inflationary taxes, cuts in productive public investment). An alternate explanation comes from Calvo (1998) who links the debt-growth issue to capital flight. The author argues that as a high distortionary tax burden on capital is required for debt servicing, capital has (or will have) a lower return and therefore there will be lower investment and growth, which leads to capital flight.

In this context, foreign debt plays a significant role. In particular, it acts like a tax when there is any improvement in the economic performance of the indebted country, as part of the gains go to higher debt repayments, that is, creditors receive part of the fruits of increased production and/or exports by the debtor country (Karagöl & Bilimler, 2004).

In the relevant literature, there have been a number of empirical studies that indicate a negative association between debt and growth for developing countries.[2] In example, Cunningham (1993) examines the relationship between the level of (total) debt and economic growth for sixteen heavily indebted nations and concluded that the former has a negative effect on the later as the productivity of capital and labor are significantly reduced.[3] Sachs (1986) argues that when indebted countries pay their debt, these payments require a transferring of resources from the private sector to the public sector. Feldstein (1986) furthermore argues that servicing external debt places

[2]For a detailed survey of the relevant empirical literature see Karagöl & Bilimler (2004) and Maier (2005).

[3]However, the author finds this result only for the 1971-1979 period. For the 1980-1987 period there is no such indication. This may be so as growth for these countries remained at a very low level due to the debt overhang from the previous decade.

pressures on foreign exchange reserves.

A particular aspect that has also concerned the literature are the effects of external debt on the economic growth rates for developing countries. Cunningham (1993) and Deshpande (1997) find a strong negative relationship between external debt and economic growth for developing countries. Sawada (1994) and Rockerbie (1994) indicate that external debt leads to a decrease in investment and economic growth. Pattillo et al. (2002) find that the average impact of external debt on per capita GDP growth is negative only for high levels of debt.[4] Afxentiou (1993) examined the negative impact of foreign indebtedness on the growth of GDP for twenty developing countries for the 1971-1988 period. The author concludes that in seven out of twenty countries the debt service ratio (total debt service to exports of goods and services) seems to be as a growth suppressing factor, while in six out to twenty, the interest service ratio (total interest payments to exports of goods and services) was found to be the most significant factor. However, Pattillo et al. (2002) and Afxentiou & Serletis (1999) have concluded that there is no causal relationship between GDP growth and foreign debt service.

An important issue is, given that high levels of indebtedness (especially external) hamper economic growth, what actions could governments and international organizations undertake to stimulate economic growth especially in developing countries. One proposed solution is for governments to imple-

[4] In particular, this result holds for for net present value of debt levels above 160%-170% of exports and 35%-40% percent of GDP. The authors findings suggest that doubling debt levels slows down annual per capita growth by about half to a full percentage point. However, they do not find a relationship between debt servicing and economic growth.

ment medium term strategies that would lead to a reduction of the level of debt (especially foreign held) in absolute terms. However, one should not expect a reduction of external debt to spontaneously transgress in an increase in private investments, especially foreign direct investments (FDI). Potential investors may be concerned that governments may once again resort to extensive external borrowing after these private investments are realized and/or heavily tax the results of these investments in order to further decrease the level of the external debt. Thus, a moral hazard problem arises. Furthermore, many of these highly indebted countries do not have the proper (public capital) infrastructure to attract productive private investments, especially FDI.

Another suggestion that is especially popular in the mass media is for external debt relief for developing nations, perhaps conditional on structural changes aiming to reduce poverty. This debt relief is proposed on the basis that external debt servicing is so high and accumulative for some countries that it is not possible for them to escape poverty. However, this proposition also leads to a moral hard issue as the possibility of repeated implementation will wary future (business and finance) lenders and investors.[5]

In this paper we focus on the effect of the internal-external debt ratio (and thus the internal-external debt servicing) on economic growth, that is,

[5]Obviously, exceptions can be made for extreme circumstances. In example, after a devastating earthquake hit Haiti on 12 January 2010, there have been many debt relief initiatives to eliminate its external debt which was $1.27 bil. in 2009 (19% of GDP). In particular, according to the IMF, the Heavily Indebted Poor Countries (HIPC) initiative has pledged $265 mil. and the Multilateral Debt Relief Initiative (MDRI) $972.2 mil.

whether lower levels of external debt (relative to internal debt) affect the long run growth rate. In particular, we find that by substituting internal for external debt higher long term economic growth rates are achieved. We also find that the speed of transition along the balanced growth path increases and that there is an optimal tax rate. The intuition is that with internal debt, the tax revenues on the interest rates from government bonds held locally can be (at least partially) utilized for productive public investments.[6] These investments act as a positive externality on production and thus increase the productivity of capital and labor, therefore providing a sustainable solution for higher growth rates with no issues of moral hazard or extraordinary costs (e.g. for international organizations). In other words, by substituting external for internal borrowing, there will be a smaller transfer of financial resources abroad (through debt servicing),[7] and marginal taxation can be used in a productive manner without disrupting (domestic and international) financial markets. This is consistent with Hofman & Reisen (1991) who argue that there is no debt overhang in debtor countries, where the transfers of financial resources from debtor countries to the other countries are a more important explanation for the investment reduction than levels of debt outstanding. Furthermore, Cohen (1993) showed that although the level of debt was not unconditionally associated with GDP growth rates for developing countries in the 1980s, the actual service of the debt crowded out

[6] For a detailed survey on productive government expenditure and economic growth see Irmen & Kuehnel (2009).

[7] This "'transfer"' problem was also highlighted by Keynes (1929).

investment.[8]

In the relevant theoretical literature, Brauninger (2005) examines a model where the budget-deficit ratio is fixed and shows the existence of a critical level for a steady state to exist. The author argues that below this critical level there are two steady states where capital, output and public debt grow at the same rate. In a similar paper, Saint-Paul (1992) analyzes public debt in an overlapping generations model with endogenous growth. In particular, Saint-Paul (1992) assumes an AK technology process and argues that the government has to adjust the tax to maintain a fixed debt-output ratio. Josten (2000) also examines an overlapping generations model with human capital formation and arrives at similar conclusions with Saint-Paul (1992).

Diamond (1965) argued that external debt has two effects in the long run. First, debt servicing taxes directly reduce the available lifetime consumption of the individual taxpayer, and second, by reducing his disposable income, taxes reduce his savings and thus the overall capital stock. The author furthermore notes that: "...internal debt has both of these effects as well as a further reduction in the capital stock arising from the substitution of government debt for private capital in individual portfolios ...".

This paper focuses on how the ratio of internal to external public debt affects the long run level of economic growth. The analysis is based on an endogenous economic growth model such as those of Lucas (1988) and Romer (1986). In particular, we assume an AK production structure that

[8]Cohen (1993) found that 1 percent of GDP paid abroad reduced domestic investment by 0.3 percent of GDP, which was identical to the correlation between investment and foreign finance observed in the 1960s.

is positively affected by an externatily which is "'productive"' public capital (Barro, 1990).

We assume that the internal-external public debt ratio and the public-private capital ratio are fixed so as to examine their effects on the long run economic growth rate. Using numerics, our analysis shows that lower shares of external public debt (relevant to internal debt) leads to higher long run growth rates. However, we must note that in a different context Panizza (2008) warns that domestic borrowing is also accompanied by many adversities (e.g. the public sector crowding out the private sector, domestic banks holding on to too much public debt therefore threatening financial stability).

The rest of the paper is organized as follows: Section 2 presents the model. Section 3 analyzes the equilibrium. In Section 4 we resort to a numerical solution to examine the effects of a change in the ratio of external to internal public debt on the long run economic growth rate. Finally, section 5 summarizes the results and provides some suggestions as to how internal government bond markets can be developed.

2. The Model

We examine a competitive decentralized model with three sectors: households, firms and the government. Households consume goods which are produced by firms, supply labor to these firms and allocate their wealth between two assets. These assets are private capital which is rented to firms at the real interest rate (r_t), and public debt.[9] Firms use labor and private capital and maximize their profits in a competitive goods market. In addition, production is also affected by a positive externality which is productive public capital. The government raises taxes and invests in public capital, services existing government debt and issues new debt according to its dynamic budget constraint, which is in effect a public debt accumulation equation.

We proceed to examine the structure of our model. First, we consider production. As noted, firms use labor and private capital in the production process which is also (positively) affected by an externality, public capital. For simplicity, the units of the population are normalized to one and the rate of growth of the population is assumed to be zero as to avoid issues of scale effects. The production function is expressed in terms of constant labor units, is a Cobb-Douglas function with constant returns to scale in private and public capital. We assume that there is a large number of identical firms and therefore we can aggregate the production process to a single AK type function as follows:

$$Y_t = AK_t^{(1-a)}G_t^a \tag{1}$$

where K_t, G_t are the total stock of private capital and public capital respectively at time t, a $(0 < a < 1)$ is a constant that expresses the output elasticities of each input and A represents total factor productivity which is normalized to one ($A = 1$). We assume capital and labor markets are

[9] We assume that public debt is issued through financial attainable and divisible financial assets (i.e. bonds).

perfectly competitive.

As labor supply (units of population) is normalized to one and the rate of growth of the population is assumed to be zero, the real wage (ω_t) and the real interest rate (r_t) are derived from the firm's optimization problem (eq. 1):

$$\omega_t = aK_t^{1-a}G_t^a \tag{2}$$

$$r_t = (1-a)K_t^{(-a)}G_t^a \tag{3}$$

We now turn our focus to the households. For simplicity, we assume an infinitely lived representative household. The present value of the utility of the representative household is given by the following equation:

$$\max_{C_t} U = \int_0^{\infty} e^{-\rho t}u(C_t)dt = \int_0^{\infty} e^{-\rho t}\frac{C_t^{1-\sigma}}{1-\sigma}dt \tag{4}$$

where C_t is private consumption at time t, $\rho > 0$ is the discount factor and $\sigma > 0$ is the inverse elasticity of substitution.

The aggregate savings (wealth) of the household are used to fund private capital capital (K_t) and (internally held) public debt (B_t^{int}) which is a subset of total public debt (B_t). We assume that there is no capital depreciation and that the government only taxes the households income from wealth (e.g. financial income). The government's dynamic constraint can be written as follows:

$$\dot{B}^{int} + \dot{K} = (1-\tau)r(K_t + B_t^{int}) - C_t + \omega_t \tag{5}$$

where the dot over the variables denote the time derivative ($\dot{B}^{int} = dB^{int}/dt$), τ is the tax rate and r is the (international) interest rate.

The accumulation of public investments (public capital) is expressed through the following equation:

$$\dot{G} = \phi\tau r(K_t + B_t^{int}) \tag{6}$$

where ϕ is the fixed share of tax revenues that are allocated for public investments (see Ghosh & Mourmouras, 2004).

The intertemporal optimum problem of the households requires the maximization of the following present value Hamiltonian function:

$$H_t = \frac{C_t^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} + q_t[(1-\tau)r(K_t + B_t^{int}) - C_t + \omega_t] \tag{7}$$

where q_t is the dynamic Langrangian multiplier. The representative agent chooses C_t, K_t, and B^{int} to maximize (7) and this leads to the following first-order conditions:

$$\frac{\partial H_t}{\partial C_t} = 0 \Rightarrow -q_t + e^{-\rho t} C_t^{-\sigma} = 0 \tag{8}$$

$$\frac{\partial H_t}{\partial K_t} = -\dot{q} \Rightarrow q_t r_t (1-\tau) = -\dot{q} \tag{9}$$

$$\frac{\partial H_t}{\partial B_t^{int}} = -\dot{q} \Rightarrow q_t r_t (1-\tau) = -\dot{q} \tag{10}$$

$$\lim_{t\to\infty} q_t e^{-\rho t}(K_t + B_t^{int}) = 0 \tag{11}$$

If the transversality condition is satisfied (eq. 11), then equations (8)-(10) are necessary and sufficient conditions for maximization.

The public sector completes the model. The role of the government is to invest in productive public capital which is utilized by the firms in the production process as a positive externality. In effect, public capital can be regarded as a device that increases productivity. In order to fund this investment in public capital and to service the existing debt (i.e. interest payments), the government can raise taxes and capital (e.g. sell bonds) in local and in international markets.[10] Therefore, the debt accumulation function can be written as:

$$\dot{B} = r_t B_t - T_t + G_t \tag{12}$$

where $\dot{B}$ is the accumulation of public debt, $r_t B_t$ is the debt servicing cost and T_t are total taxes raised by the government.

Taking into account that the government only taxes the households incomes from assets, we can rewrite the debt accumulation function (12) as follows:

$$\dot{B} = r_t B_t - \tau r_t (K_t + B_t^{int}) + G_t \tag{13}$$

We assume that the economy is closed (no commodity trading) but the government has access to international financial markets to raise capital for spending (public capital) and debt servicing (interest rate payments). This

[10]We assume that only the government has access to international financial markets. This is a realistic assumption for many developing countries.

assumption enables us to isolate the effects in changes of the internal-external debt ratio. Total debt (B) consists of internal (B^{int}) and external debt (B^{ext}). We denote the internal-external public debt ratio with ψ ($B_t^{int}/B_t^{ext} = \psi$). As the purpose of the paper is to isolate the effects of different levels of the internal-external debt ratio on the economic growth rate, we consider the internal-external public debt ratio and the public-private debt ratio to be exogenous.

Using our notation for the internal-external debt ratio we can rewrite the debt accumulation equation (13) as:

$$\dot{B} = r_t B_t - \tau r_t \left(K_t + \frac{\psi}{1+\psi} B_t + G_t \right) \tag{14}$$

where $B_t^{int} = \psi/(1+\psi)B_t$ and $B_t^{ext} = 1/(1+\psi)B_t$

3. Equilibrium

We now proceed to characterize the equilibrium of the model. From the maximization conditions of the household, especially equations (9) and (10), we find that the optimal interest rate of (internal) public debt (B_t^{int}) is equal to the real interest rate of the economy (r_t), which is the marginal product of private capital.

This equilibrium of the products of the two different assets provide the non-arbitrary condition, which in conjunction with the transversality condition (eq. 11), excludes Ponzi-type games from occurring in the economy.

Taking the first differential of (8) for time and then (natural) logs, we find the consumption dynamics as follows:

$$\frac{\dot{C}}{C} = -\frac{\rho}{\sigma} - \frac{1}{\sigma}\frac{\dot{q}}{q} \tag{15}$$

Combining equations (15) and (9) we have:

$$\frac{\dot{C}}{C} = -\frac{\rho}{\sigma} + \frac{r_t(1-\tau)}{\sigma} \tag{16}$$

Combining (16) with our equation for the real interest rate (3) we find:

$$\frac{\dot{C}}{C} = -\frac{\rho}{\sigma} + \frac{(1-a)G_t^a K_t^{-a}(1-\tau)}{\sigma} \tag{17}$$

In a similar fashion and taking into account the government's public investment equation (6), we arrive at the following expressions regarding the dynamics of the level of debt, public investments and private capital:[11]

$$\dot{B} = rB - r\tau\left(\frac{\psi B}{1+\psi} + K\right) + \tau\varphi(rB^{int} + rK) \tag{18}$$

$$\frac{\dot{G}}{G} = (\tau\varphi)(1-a)G^{-1+a}K^{1-a}\left(1 + \frac{\psi}{1+\psi}\right) \tag{19}$$

$$\dot{K} = -C + aG^aK^{1-a} + (1-a)(1-\tau)G^aK^{-a}\left(\frac{\psi B}{1+\psi} + K\right) \tag{20}$$

As we have assumed zero population growth and a fixed technological process the long run equilibrium will be characterized by long run stable growth, which we denote by γ. Our balanced growth path can be characterized by:

$$\frac{\dot{Y}}{Y} = \frac{\dot{C}}{C} = \frac{\dot{B}}{B} = \frac{\dot{B}^{int}}{B^{int}} = \frac{\dot{B}^{ext}}{B^{ext}} = \frac{\dot{G}}{G} = \frac{\dot{K}}{K} = \gamma \tag{21}$$

[11] The full algebra is available from the authors upon request.

4. Numerical Solution

In order to examine the effects of different levels of the internal-external public debt ratio (ψ) on the long run rate of economic growth (γ^*), we resort to a numerical analysis. After assuming values for some basic parameters,[12] we find the growth rate for different values of ψ.

In Figure 1 we illustrate the relationship between the different levels of the internal-external debt ratio (ψ) and the long term economic growth rate (γ). From this Figure it is clear that a higher rate of internal (as to external) debt leads to a higher rate of economic growth. In other words, for the same level of public debt, substituting external with internal debt leads to an increase in the stock of productive public and therefore productivity and output.[13]

[INSERT FIGURE 1 SOMEWHERE HERE]

Our results lead to two primary conclusions. First, an increase in the level of the public debt (in per private capital terms) has a negative effect on the economic growth rate. This is consistent with most of the existing literature (eg. Modigliani, 1960; Saint-Paul, 1992; Brauninger, 2005). Second, as the ratio of internal to external public debt increases, there is a reduction in the long run rate of economic growth.

The intuition behind these results is as follows: an increase in the ratio of internally to externally held debt leads to a decrease of private capital as

[12] The values we set are: $\sigma = 2$, $\rho = 0.3$, $\tau = 0.3$, $a = 0.25$ and $\phi = 0.3$. We increment ψ by 0.1 from 0.1 to 2. Our results are quite robust to changes in these values.

[13] The exact relationships between these variables are non-linear.

households invest more in public debt (Diamond, 1965). However, this will also increase the tax base as more households hold public debt which income (interest rates) are taxed. The taxes are used for the creation of public capital which increases productivity and therefore will lead to an increase in the long run equilibrium economic growth rate.

We now proceed to examine the effect of a change in the internal-external debt ratio (ψ) on the speed of transition towards the steady state, that is, on the balanced growth path. After some algebra we derive the law of motion (e.g. for g) as:[14]

$$\begin{aligned}
&\frac{-g\rho^2(1+\psi)+(-1+a)g^a\rho\sigma\tau\varphi(1+\psi)-(-1+a)^2g^{2a}\sigma\tau(-1+\sigma+\tau)\varphi(1+\psi)}{g\sigma\left[-\rho(1+\psi)+(-1+a)g^a\delta\right]}-\\
&+\frac{(-1+a)^2g^{1+2a}(-1+\tau)\delta}{g\sigma\left[-\rho(1+\psi)+(-1+a)g^a\delta\right]}+\\
&+\frac{(-1+a)g^{1+a}\rho\left[2(-1+\tau)(1+\psi)+\sigma(1+(1+\tau(-1+\varphi))\psi)\right]}{g\sigma\left[-\rho(1+\psi)+(-1+a)g^a\delta\right]}
\end{aligned} \tag{22}$$

where $\delta=\psi\left[-1+\sigma+\tau+\psi(-1+\tau+\sigma(1+\tau(-1+\varphi)))\right]$

Due to the non-linearities of this expression, we resort to a numerical comparison. In particular, after assuming values for some basic parameters,[15] we examine the transitional dynamics for three different values of ψ using equation (22). Our results are illustrated in Figure 2 where we see that as the ratio of external debt decreases, the speed of transition to the steady

[14] The full algebra is available from the authors upon request.

[15] The values we set are: $\sigma = z$, $\rho = 0.3$, $\tau = 0.3$, $a = 0.25$ and $\phi = 0.3$ We set $\psi = \{0.5, 0.55, 0.6\}$.

state increases.[16]

[INSERT FIGURE 2 SOMEWHERE HERE]

Finally, we conduct an exercise to derive optimal taxation. Using numerics we find that the effect of taxes on the long run economic growth rate follows a bell-shape curve (Figure 3). Therefore, there is an optimal (non zero, non unity) tax rate that maximizes economic growth. In other words, (implicitly) there exists an optimal mix of public and private capital that maximizes growth.

[INSERT FIGURE 3 SOMEWHERE HERE]

5. Conclusions

Summarizing, the level of economic growth depends on both the internal-external debt and on the public-private capital ratios. Examining the steady state we find that even a partial switch from external to domestic borrowing will lead to a reduction of capital outflows. The savings are (partially) reinvested directly for private capital and indirectly for public capital (through taxes) by the households, which in turn lead to an increase of the long run economic growth rate. We also find that an increase in internal debt (relative to external debt) also increases the speed of transition along the balanced growth path. Finally, we find that there is an optimal tax rate that maximizes growth, that is, taxes follow a Laffer-type curve.

[16]However, the steady state equilibrium requires more time periods to achieve as we now reach a higher steady state growth rate.

However, the practical issue on how a developing country can create a sizable local market for its government bonds remains. International organizations (e.g. IMF, OECD) can provide a vital role in this process through various actions (apart from the obvious of reducing corruption and increasing the transparency and effectiveness of public administration). In particular, they may organize a transfer of know-how and technical expertise on creating and maintaining credible Public Debt Management (PDM) agencies (e.g. issues of transparency and independence).[17] However, as local and international investors often view government institutions with skepticism especially in developing countries, it would be useful to actually provide some sort of accreditation from a respectable international organization to those PDM agencies that fulfil the relevant criteria. Another important step would be to develop effective Electronic Trading Platforms (ETP) for trading bonds. E.g. an African-wide ETP similar to the EuroMTS trading platform could be developed for large bond issues. Finally, the relevant literature has shown that there is a strong home bias in bond and equity markets when the real exchange rate volatility is low. In particular, Fidora et al. (2006) show that the home bias is stronger in assets with low local currency return volatility in both industrialised and emerging market countries.[18] Strategies to reduce the real exchange rate volatility (e.g. creating monetary unions like the proposed one in the South Africa region) would also boost both investor

[17]For a detailed study on the PDM guidelines and their macroeconomic effects see Currie et al. (2003).

[18]The authors show that a reduction of the monthly real exchange rate volatility from its sample mean to zero reduces bond home bias by up to 60 percentage points.

confidence and provide other positive externalities such as increases in local private export-based investments.

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Figure 1: Plot of the long run economic growth rate (γ) for different levels of the internal-external debt ratio (ψ).

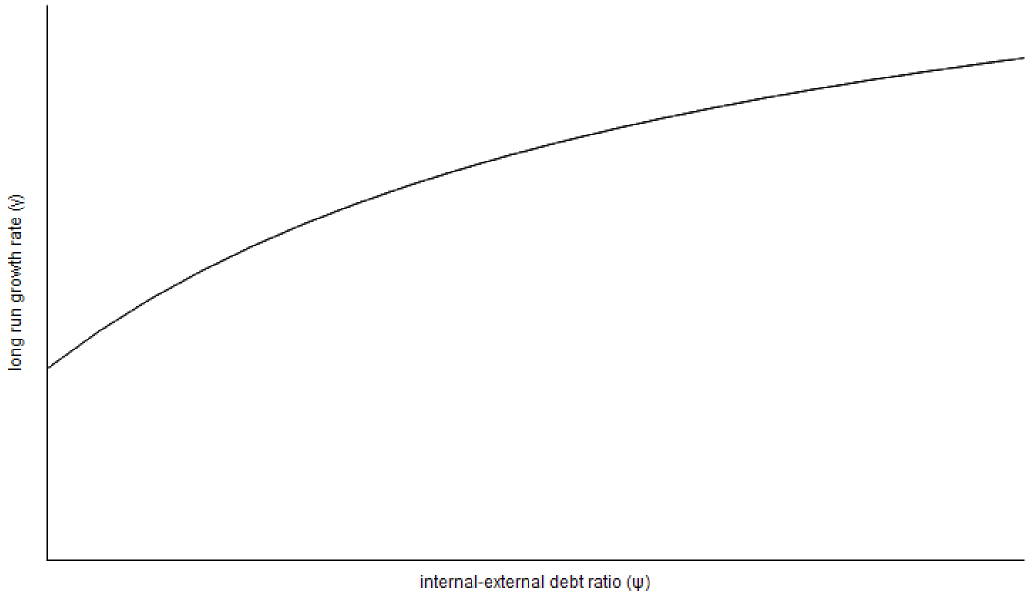
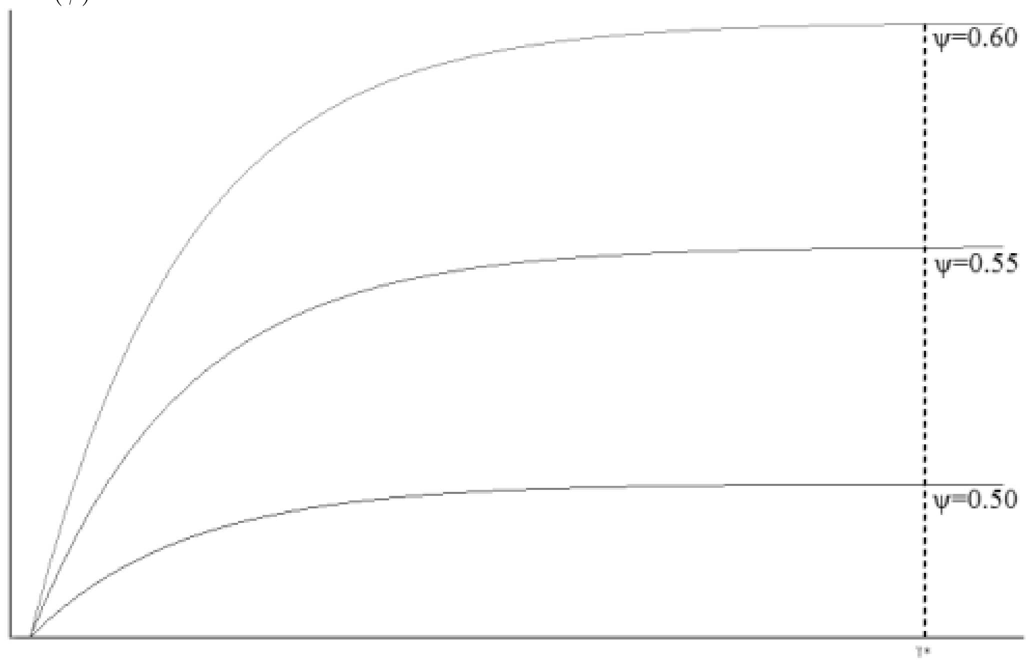
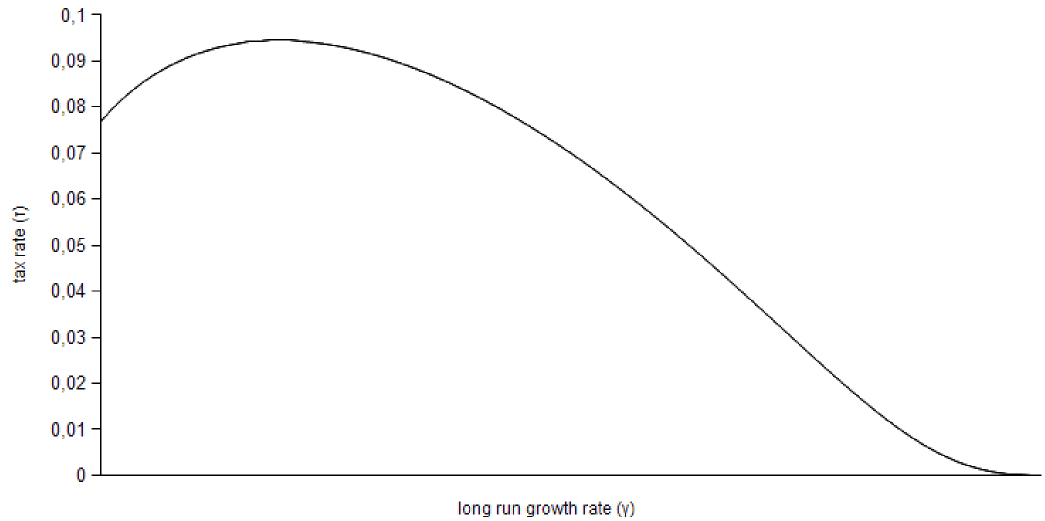


Figure 2: Plot of balanced growth paths for different levels of the internal-external debt ratio (ψ).



Note: γ^* is the steady-state.

Figure 3: Plot of the tax rate (τ) over the long run growth rate (γ).



Appendix Appendix A The algebra (Not for publication)

Appendix A.1 Firms

Production is charecterized by the following function:[19]

$$Y = K^{1-a}G^{a} \tag{A.1}$$

Soilving the firm's optimization problem we retrieve the wage (ω) and the interest rate (r):

$$\omega = aK^{1-a}G^{a} \tag{A.2}$$

$$r = (1-a)K^{-a}G^{a} \tag{A.3}$$

Appendix A.2 Public capital

The accumulation of public capital (investments) is:

$$\dot{G} = \varphi\tau(rK + rB^{int}) \tag{A.4}$$

Appendix A.3 Households

The present value of the utility of the (infinitely lived) representative household is given by:

$$\max_{C} U = \int_{0}^{\infty} e^{-\rho t}u(C)dt = \int_{0}^{\infty} e^{-\rho t}\frac{C^{1-\sigma}}{1-\sigma}dt \tag{A.5}$$

The intertemporal optimum problem of the households requires the maximization of the following present value Hamiltonian function:

[19]Time scripts have been ignored to reduce clutter.

$$H = e^{-\rho t}\frac{C^{1-\sigma}-1}{1-\sigma} + q((1-\tau)r(K+B) + \omega - C) \tag{A.6}$$

which can be written as:

$$H = q(v + (B+K)r(1-\tau) - C) + \frac{e^{-t\rho}\left(-1 + C^{1-\sigma}\right)}{1-\sigma} \tag{A.7}$$

The maximazation process gives us:

$$\frac{\partial H}{\partial C} = -q + e^{-t\rho}C^{-\sigma} = 0 \tag{A.8}$$

$$\frac{\partial H}{\partial K} = qr(1-\tau) = 0 \tag{A.9}$$

$$\frac{\partial H}{\partial B} = qr(1-\tau) = 0 \tag{A.10}$$

After some algebra we arrive at the following expression:

$$\frac{\dot{C}}{C} = -\frac{\rho}{\sigma} + \frac{(1-a)g^{a}(1-\tau)}{\sigma} \tag{A.11}$$

where $g = G/K$.

Appendix A.4 Debt

Total debt (B) is the sum of internal (B^{int}) and external debt (B^{ext}):

$$B = B^{int} + B^{ext} \tag{A.12}$$

We denote the internal-external debt ratio with ψ:

$$\frac{B^{int}}{B^{ext}} = \psi \tag{A.13}$$

The above can also be written as:

$$B^{int} = \frac{\psi B}{1+\psi} \tag{A.14}$$

$$B^{ext} = \frac{B}{1+\psi} \tag{A.15}$$

Appendix A.5 Government

$$\dot{B} = rB + G - r\tau(B^{int} + K) \tag{A.16}$$

Combining the above equation with A.14:

$$\dot{B} = rB + G - r\tau\left(\frac{\psi B}{1+\psi} + K\right) \tag{A.17}$$

Appendix A.6 Equilibrium

We now proceed to find the differential equations that charectirize the equilibrium.

Appendix A.6.1 Consumption dynamics

As found previously (A.11), the dynamics of consumption can be summarized as:

$$\frac{\dot{C}}{C} = -\frac{\rho}{\sigma} + \frac{(1-a)g^a(1-\tau)}{\sigma} \tag{A.18}$$

Appendix A.6.2 Debt dynamics

We have found:

$$\dot{B} = rB + G - r\tau\left(\frac{\psi B}{1+\psi} + K\right) \tag{A.19}$$

Combining the above with (A.4) gives us:

$$\dot{B} = rB - r\tau\left(\frac{\psi B}{1+\psi} + K\right) + \tau\varphi(rB^{int} + rK) \tag{A.20}$$

Combining the above with (A.14) and (A.3) gives us:

$$\begin{aligned}\dot{B} =&(1-a)BG^aK^{-a} - (1-a)\tau G^aK^{-a}\left(\frac{\psi B}{1+\psi} + K\right) + \\ &+ \tau\varphi\left((1-a)G^aK^{1-a} + \frac{(1-a)\psi BG^aK^{-a}}{1+\psi}\right)\end{aligned} \tag{A.21}$$

We use the following notations to express our variables in per private capital terms: $b = B/K$, $c = C/K$ and $g = G/K$. Using the above equation we create the expression for the dynamics of public debt ($\dot{B}/B$):

$$\frac{\dot{B}}{B} = -\frac{(-1+a)g^a(\tau(-1+\varphi)(1+\psi) + b(1+(1+\tau(-1+\varphi))\psi))}{b(1+\psi)} \tag{A.22}$$

Appendix A.6.3 Public capital dynamics

We combine equations (A.4) and (A.14), take the derivative for time and arrive at the following expression:

$$\frac{\dot{G}}{G} = \frac{\tau\varphi\left(\frac{r\psi B}{1+\psi} + rK\right)}{G} \tag{A.23}$$

We replace r in the above equation (eq. A.3):

$$\begin{aligned}\frac{\dot{G}}{G} =&\tau\varphi G^{-1+a}K^{1-a} - a\tau\varphi G^{-1+a}K^{1-a} + \\ &+ \frac{\tau\varphi\psi BG^{-1+a}K^{-a}}{1+\psi} - \frac{a\tau\varphi\psi BG^{-1+a}K^{-a}}{1+\psi}\end{aligned} \tag{A.24}$$

After some albebra we arrive at our final expression for the public capital dynamics:

$$\frac{\dot{G}}{G} = g^{-1+a}\tau\varphi - ag^{-1+a}\tau\varphi + \frac{bg^{-1+a}\tau\varphi\psi}{1+\psi} - \frac{abg^{-1+a}\tau\varphi\psi}{1+\psi} \tag{A.25}$$

Appendix A.6.4 Private capital dynamics

The dynamics of private capital is as follows:

$$\dot{K} = (1-\tau)r(K + B^{int}) + \omega - C \tag{A.26}$$

We replace the expressions for r (A.3), ω (A.2) and B^{int} (A.14) in the above equation and find:

$$\dot{K} = -C + aG^a K^{1-a} + (1-a)(1-\tau)G^a K^{-a}\left(\frac{\psi B}{1+\psi} + K\right) \tag{A.27}$$

After substitution C, B, G and conducting some algebra, we arrive to the following equation:

$$\frac{\dot{K}}{K} = \frac{-c(1+\psi) + g^a(1 + (1+b-ab)\psi + (-1+a)\tau(1+\psi+b\psi))}{1+\psi} \tag{A.28}$$

Appendix A.7 Balanced growth paths

In summary, our four differential equations that charectirize the equilibrium are:

$$\frac{\dot{C}}{C} = -\frac{\rho}{\sigma} + \frac{(1-a)g^a(1-\tau)}{\sigma} \tag{A.29}$$

$$\frac{\dot{B}}{B} = -\frac{(-1+a)g^a(\tau(-1+\varphi)(1+\psi) + b(1+(1+\tau(-1+\varphi))\psi))}{b(1+\psi)} \tag{A.30}$$

$$\frac{\dot{G}}{G} = g^{-1+a}\tau\varphi - ag^{-1+a}\tau\varphi + \frac{bg^{-1+a}\tau\varphi\psi}{1+\psi} - \frac{abg^{-1+a}\tau\varphi\psi}{1+\psi} \tag{A.31}$$

$$\frac{\dot{K}}{K} = \frac{-c(1+\psi) + g^a(1 + (1+b-ab)\psi + (-1+a)\tau(1+\psi+b\psi))}{1+\psi} \tag{A.32}$$

This is a system of four equations with four unknowns (4x4). We proceed to reduce our problem to that of 3x3 by expressing all our equations in terms of per private capital (lower case variables express the terms in per private capital).

$$\frac{\dot{c}}{c} = \frac{\dot{C}}{C} - \frac{\dot{K}}{K} \tag{A.33}$$

$$\begin{aligned} \frac{\dot{c}}{c} =& -\frac{\rho}{\sigma} + \frac{(-1+a)g^a(-1+\tau)}{\sigma} + \frac{c(1+\psi)}{1+\psi} - \\ &- \frac{g^a(1+(1+b-ab)\psi + (-1+a)\tau(1+\psi+b\psi))}{1+\psi} \end{aligned} \tag{A.34}$$

$$\frac{\dot{b}}{b} = \frac{\dot{B}}{B} - \frac{\dot{K}}{K} \tag{A.35}$$

$$\begin{aligned} \frac{\dot{b}}{b} =& \frac{-(-1+a)b^2g^a(-1+\tau)\psi - (-1+a)g^a\tau(-1+\varphi)(1+\psi) + bc(1+\psi)}{b(1+\psi)} + \\ &+ \frac{g^a(\tau + \tau\varphi\psi - a(1+\tau+\psi+\tau\varphi\psi))}{b(1+\psi)} \end{aligned} \tag{A.36}$$

$$\frac{\dot{g}}{g} = \frac{\dot{G}}{G} - \frac{\dot{K}}{K} \tag{A.37}$$

$$\frac{\dot{g}}{g} = \frac{-g^a((-1+a)\tau\varphi(1+\psi+b\psi) + g(1+(1+b-ab)\psi+(-1+a)\tau(1+\psi+b\psi)))}{g(1+\psi)} + \\ + \frac{cg(1+\psi)}{g(1+\psi)} \tag{A.38}$$

The above system of equations (3x3) characterize the balanced growth paths. Due to the non-linearities of this expression, we resort to a numerical solution (see main text).

Appendix A.8 Steady state

We now proceed to charecterize the steady state. First we set eq. (A.34) equal to zero and solve for c:

$$c = \frac{\rho}{\sigma} - \frac{(-1+a)g^a(-1+\tau)}{\sigma} + \frac{g^a(1+(1+b-ab)\psi+(-1+a)\tau(1+\psi+b\psi))}{1+\psi} \tag{A.39}$$

We substitute the above (A.39) into our equation for the debt dynamics (A.36):

$$\frac{\dot{b}}{b} = \frac{(1-a)b^2g^a(-1+\tau)\psi - (-1+a)g^a\tau(-1+\varphi)(1+\psi)}{b(1+\psi)} + \\ + \frac{b\left(g^a(\tau+\tau\varphi\psi - a(1+\tau+\psi+\tau\varphi\psi))\right)}{b(1+\psi)} + \\ + \frac{b((1+\psi)\left(\frac{\rho}{\sigma} - \frac{(-1+a)g^a(-1+\tau)}{\sigma} + \frac{g^a(1+(1+b-ab)\psi+(-1+a)\tau(1+\psi+b\psi))}{1+\psi}\right))}{b(1+\psi)} \tag{A.40}$$

We substitute (A.39) into our equation for public capital dynamics (A.38):

$$
\begin{aligned}
\frac{\dot{g}}{g} =& \frac{-g^a((-1+a)\tau\varphi(1+\psi+b\psi)+g(1+(1+b-ab)\psi}{g(1+\psi)}+ \\
&+\frac{(-1+a)\tau(1+\psi+b\psi)))+g(1+\psi)\left(\frac{\rho}{\sigma}-\frac{(-1+a)g^a(-1+\tau)}{\sigma}+\frac{g^a(1+(1+b-ab)\psi+(-1+a)\tau(1+\psi+b\psi))}{1+\psi}\right)}{g(1+\psi)}
\end{aligned}
\tag{A.41}
$$

We solve $\dot{b}/b$ for b and find:

$$
b=-\frac{(-1+a)g^a\sigma\tau(-1+\varphi)(1+\psi)}{-\rho(1+\psi)+(-1+a)g^a(-1+\sigma+\tau+(-1+\tau+\sigma(1+\tau(-1+\varphi)))\psi)}
\tag{A.42}
$$

Now we replace the above into our expression for $\dot{g}/g$ and we derive the law of motion of g:

$$
\begin{aligned}
&\frac{-g\rho^2(1+\psi)+(-1+a)g^a\rho\sigma\tau\varphi(1+\psi)-(-1+a)^2g^{2a}\sigma\tau(-1+\sigma+\tau)\varphi(1+\psi)}{g\sigma\left[-\rho(1+\psi)+(-1+a)g^a\delta\right]}- \\
&-\frac{(-1+a)^2g^{1+2a}(-1+\tau)\delta}{g\sigma\left(-\rho(1+\psi)+(-1+a)g^a\delta\right)}+ \\
&+\frac{(-1+a)g^{1+a}\rho\left[2(-1+\tau)(1+\psi)+\sigma(1+(1+\tau(-1+\varphi))\psi)\right]}{g\sigma\left(-\rho(1+\psi)+(-1+a)g^a\delta\right)}
\end{aligned}
\tag{A.43}
$$

where $\delta=\psi\left[-1+\sigma+\tau+\psi(-1+\tau+\sigma(1+\tau(-1+\varphi)))\right]$

Due to the non-linearities of this expression, we resort to a numerical solution (see main text).