Multiple Reserve Requirements and Equilibrium Dynamics

in a Small Open Economy♦

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Abstract

We modeled a typical Asian-crisis-economy using dynamic general equilibrium techniques and established exchange rates from nontrivial fiat-currency demands. The scope for existence of equilibria and dynamic properties are associated with the underlying policy regime. Binding multiple reserve requirements promotes stability under a floating exchange rate regime while backing the money supply acts as a stabilizer in a fixed regime.

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1. Introduction

A financial crisis could result from a reverse of the international capital market, a self-fulfilling panic bank-run scenario, or a sharp swing in the exchange rate. The following stylized facts summarize the prevalent view in studies of the Asian-crisis: 1) Increased risky-lending behavior by banks led to a boom in private borrowing. 2) The lack of sound financial structure was rooted in the process of financial and capital liberalization[1]. 3) Borrowing from foreign banks enabled a significant portion of domestic banks' lending[2]. 4) The credit crunch among foreign creditors directly impacted banks' solvency. 5) The major fluctuation of foreign exchange led to regime switching[3]. Some of these show important parallels to the 2007-2008 United States financial crises.

Given the complexity of interaction between monetary policies and alternative exchange rate regimes, our goal is to develop a model to produce the stylized view so that we can provide policy recommendations which will achieve stability in a future crisis. Emphasizing the dynamic properties and instability oriented from the model economy, this study fills the gap of the unsuitability of the traditional Diamond-Dybvig (1983) framework in an overlapping generations model and builds on Chang and Velasco's (2000a, 2000b, 2001) papers on financial fragility and exchange rate regimes.

The study of a typical Asian-crisis-economy is important because a financial crisis originating in Asia could spread around the world. Diamond and Rajan (2009, p.606) note,

[1] The extensive literature on financial liberalization can go as far as Goodhart and Delargy (1998), Kaminsky and Reinhart (1999), Lindgren et. a.l.(1999), Summers (2000), Boyd et al. (2000), Kishi and Okuda (2001), and Kaminsky (2003).

[2] See Calvo, Izquierdo, and Meija (2004), Calvo and Talvi (2005), Calvo and Talvi (2006), Calvo, Izquierdo, and Talvi(2006), Bordo (2006), and Reinhart and Rogoff (2008).

[3] According to chronology of the crisis, the floating of the baht in July 1997 in Thailand triggered the crisis. During the 1980s and the early 1990s, Indonesia, South Korea, Thailand, and Malaysia had managed floating arrangements. However, after the 1997 crises Indonesia, Korea and Thailand moved from intermediate pegs to free floating, but Malaysia turned to a very hard peg. See Table 1 for details.

"…This (the subprime crisis) is a crisis born in some ways of previous financial crises…. Clearly, the net financial savings generated in one part of the world have to be absorbed by deficits elsewhere." In addition, the interaction among developed and emerging markets complicates what can be done to remedy crises. De Macedo (2010, p.2-3) notes that "…developed countries and emerging markets realize that they are mutually interdependent even though they rarely coordinate their national policy responses…"

To replicate a small, open economy (SOE) with a nontrivial banking system, we built a Dynamic Stochastic General Equilibrium Model (DSGE), from the micro-foundations. The comparison of two sets of policy rules under floating and fixed exchange rate arrangements is achievable in the presence of multiple reserve requirements. Binding multiple reserve requirements promotes stability under a floating exchange rate regime while backing of the domestic money supply acts as a stabilizer only in a fixed exchange rate regime.

The paper proceeds as follows. Section 2 is the literature review. Sections 3 and 4 analyze the properties of stationary and dynamic equilibria under alternative exchange rate regimes. Section 5 is the conclusion.

2. Literature

The paper studies interaction between monetary policies and exchange rate regimes in small open economies. Our paper is related to two broad strands of research, including work on the micro-foundations of banks and work on monetary policy rules and exchange rate arrangements.

The banking literature on the micro-foundations emphasizes depositors' preference shocks and possibility of multiple equilibria. This approach described by Diamond and Dybvig (D&D) in 1983 and is applied by Cooper and Ross (1998), and Ennis and Keister (2006). Chang and Velasco's work in 2000 and 2001 is of particular relevance in discussing the effects of international capital inflow. Chang and Velasco (C&V) present a multiple equilibrium model in which the maturity of bank's external debts, the level of international reserves, and the term structure of interest rates are jointly determined (2000b). C&V (2001) shows how self-fulfilling runs on domestic banks could crash the asset price boom. Coordination failure among foreign lenders could contribute to the financial crisis.

We distinguished this study from C&V (2000(a), (b), and 2001) in two ways. First, while C&V assumes money in the utility function, we introduced non-trivial demands for multiple fiat currencies. Banks must hold a fraction of their deposits as unremunerated currency reserves. Second, we used a DSGE model in an economy with an infinite horizon, as is the Overlapping Generations. Thus, we were able to compare stability and volatility from the dynamics under each exchange rate arrangement.

Our paper is also relevant to the literature on monetary policy rules and exchange rate regimes in emerging markets. Calvo and Reinhart (1999) show that fear of floating leads many emerging markets to choose capital controls rather than dollarization, but the latter is a better market-oriented option to minimize the severity of sudden stops of foreign credits. Bordo and Meissner (2006) and Bordo (2006) provide evidence that backing hard currency debt with foreign reserves reduces the likelihood of currency and banking crises. Berger (2006) studies the optimal choice of the exchange rate in a small open economy and finds that a high substitutability between home and foreign goods favors of floating exchange rate over a hard peg. Curdia (2008), on the other hand, finds that a fixed exchange rate regime performs better than a currency peg in the economy with low nominal rigidities or high elasticity of foreign demand. Devereux, Lane, and Xu (2006) study the exchange rate flexibility in implementing monetary policy and find a clear tradeoff between real stability and inflation stability under both fixed exchange rates and inflation targeting rules. Braggion, Christiano, and Roldos (2009) study the optimal monetary response to a financial crisis similar to the Asian crisis in the dynamic general equilibrium setup but focus primarily on interest rate policies and monetary transmission mechanisms.

We found that the comparison of optimal monetary policy decisions in emerging markets is rare except in the work by Curdia (2008). The optimal monetary response given exchange rate arrangements to deal with the financial instability is still an open question in the literature. Our emphasis on monetary policy rules is different. Given the complexity of interaction between policy parameters, this study evaluates monetary policy from the aspect of dynamic properties such as volatility and stability along dynamic paths, possibility of cyclical fluctuations, and endogenously-arising volatility. The goal is to find a policy which achieves stability and minimizes the welfare impact. With multiple reserve requirements and backing of the domestic money supply, we studied Diamond-Dybvig-type banks that have access to the world credit market. From the standpoint of the scope for endogenous fluctuations under alternative exchange rate arrangements, this study identifies proper monetary policy in the model economy.

3. The Model

The model consists of an infinite sequence of two-period-lived, overlapping generations. Time is discrete and indexed by t=0, 1, 2, … .

Households

A continuum of households with unit mass born at period t is young at t and old during period $t+1$. As in D&D framework, households can become impatient with probability $\lambda \in (0,1)$ or patient with probability ($1-\lambda$). Impatient households consume when young ($c_{1,t}$), while patient households consume only when old ($c_{2,t+1}$). A typical household's expected lifetime utility is:

$$E_t\left[u\left(c_{1,t},c_{2,t+1}\right)\right]=\lambda\cdot\ln\left(c_{1,t}\right)+\left(1-\lambda\right)\cdot\ln\left(c_{2,t+1}\right). \quad (1)$$

A household receives an endowment of w together with the monetary transfer τ_t when young, regardless of types. At the same time, households have access to an illiquid investment technology that yields $R>1$ at the end of $t+1$ and $0<r<1$ in the case of early liquidation at *t*. To induce self-selection and truth-telling, the following condition must hold

$$c_{2,t+1}\geq r\cdot c_{1,t}. \quad (2)$$

Monetary authority

M_t and Q_t represent the outstanding nominal stock of domestic currency, *wons*, and foreign currency, *dollars*, at *t* [4]. The monetary authority sets the rate of money growth to be $\sigma>-1$, and the money supply follows the rule

$$M_{t+1}=\left(1+\sigma\right)\cdot M_t,\quad \forall t>0, \quad (3)$$

with $M_0>0$ given[5].

Two monetary policies are noteworthy. First, the monetary authority backs the domestic money supply by holdings B_t in the form of foreign-reserve assets that yield the world interest rate $\varepsilon>1$ from *t* to *t+1*.

4 Won, the legal currency circulated in Korea, is used in the model as the unit of domestic currency.
5 Regarding the initial conditions for the dynamic, infinite-horizon economy, (1-λ) patient initial old households at *t*=0 wish to consume $c_{2,0}$ goods. This consumption is financed by distributing the initial money supplies $M_0>0$ and $Q_0>0$ equally among the patient initial old.

$$B_t = \theta \cdot \left(\frac{M_t}{e_t} \right), \tag{4}$$

where $\theta \in [0,1]$ is the constant fraction of *dollar-value* of the supply of *wons* that the central bank chooses to back. e_t denotes the number of *wons* exchanged for one *dollar*. The financial position of the government is then summarized by the budget constraint

$$\tau_t = \frac{M_t - M_{t-1}}{p_t} - \frac{B_t - \varepsilon \cdot (1+\sigma^*) \cdot B_{t-1}}{p_t^*}, \tag{5}$$

where the second terms accounts for variations in the foreign reserve position backing the domestic money supply.

Second, the central bank sets the reserve requirements. $\phi_f, \phi_d \in (0,1)$ designate the fraction of total deposits that the banks must hold as domestic and foreign currency reserves. Note that $\phi_d + \phi_f < 1$ must hold.

Financial Intermediation

Banks have access to the world credit markets by trading in several primary debt markets: early intra-period debt, $d_{0,t}$, late inter-period debt, $d_{1,t+1}$, and long-term debt, $d_{2,t+1}$. As stated in Chang and Velasco (2000b, 2001), the banks are constrained by an upper limit set by foreign banks.

$$0 < d_{0,t} + d_{2,t+1} \le f_0, \tag{6}$$

$$0 < d_{1,t+1} + d_{2,t+1} \le f_1. \tag{7}$$

$f_1 > f_0 > 0$ are exogenous time-invariant structural parameters representing the maximum amount that foreign banks are willing to lend to domestic banks.

It is assumed that all transactions take place through banks, which are inherently financial intermediaries. Young households receive $w+\tau_t$ goods when born, and banks receive these deposits and borrow $d_{0,t}+d_{2,t+1}$ goods from the rest of the world. At the same time, banks set aside the required currency reserves of $\phi_d\cdot(w+\tau_t)$ in terms of *wons* and $\phi_f\cdot(w+\tau_t)$ in the form of *dollars*; these currency reserves are deposited in the banks' reserves accounts held within the monetary authority. The banks also invest in the illiquid primary asset, k_{t+1} , which is financed by a combination of their resources and lead to the budget constraint

$$k_{t+1}\leq d_{0,t}+d_{2,t+1}+(1-\phi_d-\phi_f)\cdot(w+\tau_t)\,. \tag{8}$$

Households' types are realized at the early afternoon of *t*. Under the truth-telling constraint, households behave as the true types. Accordingly, the banks pay a total of $\lambda\cdot c_{1,t}$ goods to impatient depositors following a sequential-service constraint and repay their early intra-period debt $r_0^*\cdot d_{0t}$ to foreign banks. At the end of *t*, banks can access a loan/bail-out inter-period debt, $d_{1,t+1}$. If more funds are required, banks liquidate early the long-term investment by the amount l_t, but this is a last resort since early liquidation is costly[6]. The budget constraint that summarizes this state is given by

$$\lambda\cdot c_{1,t}+r_0^*\cdot d_{0,t}\leq r\cdot l_t+d_{1,t+1}\,. \tag{9}$$

There is no action until late in the afternoon of *t+1* when the patient households withdraw a total of $(1-\lambda)\cdot c_{2,t+1}$ from banks. By then, banks repay the amounts of the inter-period debt, $r_1^*\cdot d_{1,t+1}$, and the long-term debt, $r_2^*\cdot d_{2,t+1}$, to foreign creditors. With regard to the sources of income, banks

[6]One could think of $d_{1,t+1}$ and l_t as substitute sources of liquidity for banks, but $d_{1,t+1}$ is cheaper, since $r_0^*<R$ holds in equilibrium. If the bank were to exhaust its resources before covering all liabilities, the bank would close, and any future payments contracted by the bank would be lost.

receive the return of illiquid investment un-liquidated, $R\cdot\left(k_{t+1}-l_t\right)$ and the gross real return on their currency reserves. Patient households take reserve requirements into account when forming their expectations, reducing the likelihood of running on banks for a given set of circumstances. The resulting budget constraint is given by

$$(1-\lambda)\cdot c_{2,t+1}+r_2^*\cdot d_{2,t+1}+r_1^*\cdot d_{1,t+1}\le R\cdot(k_{t+1}-l_t)+(w+\tau_t)\cdot\left[\phi_d\cdot\left(\frac{p_t}{p_{t+1}}\right)+\frac{\phi_f}{(1+\sigma^*)}\right]. \quad (10)$$

Deposit Contract is a state-contingent consumption $\left(c_{1,t},c_{2,t+1}\right)$ which maximizes the households' expected lifetime utility eq. (1) and subject to the truth-telling (2) and constraints (6)-(10).

General Equilibrium with Floating Exchange Rates

First, the purchasing power parity $\hat{e}_t\cdot p_t^*=\hat{p}_t$ and the no arbitrage condition $R=r_2^*=r_0^*\cdot r_1^*$ hold. Second, since currencies are dominated in return, $p_t/p_{t+1}<R$ and $p_t^*/p_{t+1}^*<R$, the currency reserve requirements bind, and the demand for real money balances are determined.

Stationary Equilibria

A stationary equilibrium for this economy is defined as the set of vectors $\left(\hat{z},\hat{\tau},\hat{q},\hat{b},\hat{k}\right)\in\mathbb{R}^5$, $\left(\hat{d}_0,\hat{d}_1,\hat{d}_2\right)\in\mathbb{R}_+^3$ and $\left(\hat{c}_1,\hat{c}_2\right)\in\mathbb{R}_{++}^2$, $\hat{l}=0$ and all the conditions in the previous section are met. The core dynamic reduced-form system includes the real money balances, $\hat{z}_t$, monetary transfers, $\hat{\tau}_t$, foreign real money balance, $\hat{q}_t$, real balances of foreign asset reserves, $\hat{b}_t$ and banks' long-term investment, $\hat{k}_{t+1}$.

Dynamic Equilibria

The dynamic system includes the core dynamic system, $\left(\hat{z},\hat{\tau},\hat{q},\hat{b},\hat{k}\right)\in\mathbb{R}^5$, the foreign debt sub-system, $\left(\hat{d}_0,\hat{d}_1,\hat{d}_2\right)\in\mathbb{R}^3_+$, and the space-contingent commodities sub-system, $\left(\hat{c}_1,\hat{c}_2\right)\in\mathbb{R}^2_{++}$. The core dynamic system in equilibrium is originated from the real balances of *wons*, $\hat{z}_t$. We listed all coefficients in Appendix A.

$$\hat{z}_{t+1} = z\left(\hat{z}_t\right) = a_0 + a_1 \cdot \hat{z}_t \tag{11a}$$

$$\hat{\tau}_{t+1} = \tau\left(\hat{z}_t\right) = b_0 + b_1 \cdot \hat{z}_t \tag{11b}$$

$$\hat{k}_{t+2} = k\left(\hat{z}_t\right) = c_0 + c_1 \cdot \hat{z}_t \tag{11c}$$

$$\hat{q}_{t+1} = q\left(\hat{z}_t\right) = d_0 + d_1 \cdot \hat{z}_t \tag{11d}$$

$$\hat{b}_{t+1} = b\left(\hat{z}_t\right) = e_0 + e_1 \cdot \hat{z}_t \tag{11e}$$

Similarly, the dynamics of $\hat{d}_{2,t+1}$ is governed by

$$\hat{d}_{2,t+1} = g_0 + g_1 \cdot \hat{z}_{t-1} + g_2 \cdot \left(\frac{\hat{z}_{t-1}}{\hat{z}_t}\right) + g_3 \cdot \left(\frac{1}{\hat{z}_t}\right) \tag{12a}$$

$$d_{0,t+1} = f_0 - \hat{d}_{2,t+2} \tag{12b}$$

$$\hat{d}_{1,t+1} = f_1 - \hat{d}_{2,t+1} \tag{12c}$$

The debt-structure vector displays nontrivial dynamics, as seen in (12a-12c). Equation (12a) is a second order, nonlinear, difference equation in $\hat{z}_t$. Likewise, the properties of the pair $\left(\hat{c}_{1,t},\hat{c}_{2,t+1}\right) \gg 0$ inherit from the system of the debt-structure. The consumption by impatient and patient households are given by

$$\hat{c}_{1,t} = h_0 + h_1 \cdot \hat{d}_{2,t+1} \tag{13a}$$

$$\hat{c}_{2,t+1} = j_0 + j_1 \cdot \hat{d}_{2,t+1} + j_2 \cdot \hat{z}_{t-1} + j_3 \cdot \left(\frac{\hat{z}_{t-1}}{\hat{z}_t} \right) + j_4 \cdot \left(\frac{1}{\hat{z}_t} \right) \tag{13b}$$

Proposition 1 Under floating exchange rates,

i) It transpires that $0 < a_1 < 1$ holds and the core dynamics are always stable and monotonic. The existence of stationary equilibria depends on the values a_0; $a_0 > 0$ obtains and the condition $0 < a_1 < 1$ (i.e. stability) guarantees that the unique core-equilibrium exists for the total parameter space. The dominant eigenvalue of the system is a smooth function of the policy parameters.

(ii) We observed non-monotonic dynamics of the long-term debt asset, $\hat{d}_2$, and the monotonic dynamics of the short term debts, $\hat{d}_0$ and $\hat{d}_1$. Unstable and oscillating divergence occurs for some values of σ, causing the vanishing of equilibria.

(iii) $\hat{c}_{1,t}$ and $\hat{c}_{2,t+1}$ inherit the dynamics from $\hat{z}_t$ and $\hat{d}_{2,t+1}$. Both transpire non-monotonic dynamics with the negative roots dominant in magnitude. Thus, no endogenously-arising volatility is observed along equilibrium dynamic paths.

We report the equilibrium outcomes for a range of values of the growth of domestic money supply, σ, currency reserves, ϕ_d, and backing of the domestic money supply, θ, in a numerical example. We adopted the following parameter values in the benchmark:

$w = 2$, $\phi_d = \phi_f = 0.1$, $\theta = 0.2$, $\lambda = 0.2$, $\sigma^* = 0.05$, $\tilde{r} = 1.1$, $R = r_2^* = 1.2$, $r_0^* = 1.08$, $r_1^* = 1.11$.

The reader should note that the calibration and parameterization of the model for optimal monetary policy are beyond the scope of this study. We presented a quantitative analysis of the simulations following a reasonable set of parameters. Then, we reported the sensitive analysis for each policy parameter[7].

Let $i = 1,2,3,4,5$ index the associated eigenvalues $\hat{\epsilon}_i(\sigma)$ for the variables $\left(\hat{z}_t, \hat{\tau}_t, \hat{q}_t, \hat{b}_t^*, \hat{k}_{t+1}\right)$ in the core. $\hat{\epsilon}_i(\sigma)\ \forall i$ is a monotonically increasing and concave function of the rate of domestic money growth, σ. We observed the monotonic dynamic of the core stationary variables given a fixed combination of parameters in the benchmark.

When the monetary authority maintains backing of the domestic money supply as low as 0.1, the steady-state vector $\left(\hat{d}_0, \hat{d}_1, \hat{d}_2\right)$ is always a saddle no matter the size of the rate of domestic money growth, σ. The steady state is neither stable nor unstable. Only a specific set of parameters could reach the steady state along the saddle path. However, if backing the domestic money supply is set to the highest level, θ=1, both eigenvalues are outside the unit circle and the dynamic system is unstable as σ increases. The steady-state $\hat{d}_2$ is an oscillatory divergence with large and negative eigenvalues, and short-term debts $\hat{d}_0, \hat{d}_1$ are monotonic divergence as a source.

Thus, when the objective of the monetary authority is to fully back the domestic money supply, increases in the domestic money growth will increase the fluctuation and instability of the foreign debts. In some cases, the fluctuations can be large and explosive, ruling out the existence of equilibrium sequences, since $\hat{d}_{2,t+1}$ is bounded by f_0. Unstable and oscillating divergences occur for some values of σ, causing the vanishing of equilibria. In addition, we found that increases in reserve requirements $\left(\phi_d, \phi_f\right)$ improve the stability in the dynamic system by shifting the dynamic from a source back to a saddle.

[7] The result of simulation is available based on request from the authors.

Finally, the eigenvalues of the state-contingent consumption are smooth functions of the policy parameters. Increases in the reserve requirements in the economy that chooses to back the domestic money supply will reduce the magnitude on one of the eigenvalues drastically. Thus, a steady state becomes a saddle with non-monotonic fluctuations along the stable manifold.

4. Fixed Exchange Rates

This economy is identical to the one discussed in Section 3 in every respect except for the exchange rate regime. At period 0, the monetary authority sets both e and θ where $\theta \in [0,1]$[8].

$$B_t^* = \theta \cdot \left(\frac{M_t}{e} \right). \tag{14}$$

$$\tau_t = \frac{M_t - M_{t-1}}{p_t} - \frac{B_t^* - \tilde{r} \cdot \left(p_t^* / p_{t-1}^* \right) \cdot B_{t-1}^*}{p_t^*} = \phi_d \cdot (w + \tau_t) - (p_{t-1}/p_t) \cdot \phi_d \cdot (w + \tau_{t-1}) - \left(b_t^* - \tilde{r} \cdot b_{t-1}^* \right) \cdot \tag{15}$$

The first two terms on the right hand side of equation (15) represent the real value of any changes in the nominal supply needed to sustain the fixed exchange rate. The third term indicates the effects of any changes in the real foreign-reserve position of the government. The rate of return on the real *won* balance changes accordingly under a hard peg as

$$\left(\overline{p}_t / \overline{p}_{t+1} \right) = \left(p_t^* / p_{t+1}^* \right) = \left(1 + \sigma^* \right)^{-1}. \tag{16}$$

Equation (16) implies that σ^* is not a parameter under the control of the monetary authority, reflecting the lack of control of the domestic money supply. We must remark again that, under this hard peg, the dynamics of the system originates in τ_t instead of z_t as it was the case under floating exchange rates. The laws of motion regarding the dynamic system and the derivation of the steady-state equilibria under the fixed exchange rate regime are available in Appendix B.

[8] A currency board arrangement obtains when the monetary authority sets $\theta = 1$ once-and-for-all at $t = 0$.

Stationary Equilibria

Stationary equilibria under fixed exchange rates are defined by allocations such that $\left\{\left(\overline{\tau},\overline{z},\overline{q},\overline{b^{*}},\overline{k}\right),\left(\overline{d}_{0,j},\overline{d}_{1,j},\overline{d}_{2,j}\right),\left(\overline{c}_{1,j},\overline{c}_{2,j}\right) \mid \overline{l}=0\right\} \in \mathbb{R}_{++}^{5}\times\mathbb{R}_{+}^{3}\times\mathbb{R}_{++}^{2}$, which satisfy all the conditions given above. We analyzed the set of ***separating*** stationary equilibria that all households behave according to their true types and thus have no need of early liquidity. This second model economy, similar to the economy under a floating regime, violates standard conditions of regularity. Regarding the number of equilibria, there is typically a continuum of equilibria in this economy. The mapping between the vectors of relative prices and the demand correspondence is not unique.

Dynamic Equilibria

Under a fixed exchange rate regime, the dynamics originate from the monetary authority's budget constraint, and both the money supply and the holdings of foreign-reserve assets must adjust to keep the nominal exchange rate at its set level. The dynamic behavior is governed by the core variables $\left(\overline{\tau}_t,\overline{z}_t,\overline{q}_t,\overline{b}_t^{*},\overline{k}_{t+1}\right)$, both with respect to σ^{*} and θ. Proposition 2 summarizes the dynamic properties under a fixed exchange rate regime.

Proposition 2 Under fixed exchange rates:

Let $i=1,2,3,4,5$ index each of the core variables in $\left(\overline{\tau}_t,\overline{z}_t,\overline{q}_t,\overline{b}_t^{*},\overline{k}_{t+1}\right)$, while $\left(\overline{\epsilon}_1,\overline{\epsilon}_2,\overline{\epsilon}_3,\overline{\epsilon}_4,\overline{\epsilon}_5\right)$ denotes the vector of associated eigenvalues. For each core variable i, the eigenvalue $\overline{\epsilon}_i(\sigma^{*},\theta)$ is a monotonically increasing function of σ^{*} and θ. In particular, $\forall i$, there exists a vector of bifurcation values $\left(\hat{\sigma}_i^*, \breve{\sigma}_i^*, \overline{\sigma}_i^*\right)$, where $-1<\hat{\sigma}_i^*<\breve{\sigma}_i^*<\overline{\sigma}_i^*<\infty$, $\overline{\epsilon}_i\left(\hat{\sigma}_i^*\right)=-1$, $\overline{\epsilon}_i\left(\breve{\sigma}_i^*, \theta\right)=0$ and $\overline{\epsilon}_i\left(\overline{\sigma}_i^*, \theta\right)=1$, for σ^* and a vector $\left(\hat{\theta}_i, \breve{\theta}_i, \overline{\theta}_i\right)$ of bifurcation values of θ, such that $\hat{\theta}_i<\breve{\theta}_i<\overline{\theta}_i$, $\overline{\epsilon}_i\left(\sigma^*, \hat{\theta}_i\right)=-1$, $\overline{\epsilon}_i\left(\sigma^*, \breve{\theta}_i\right)=0$ and $\overline{\epsilon}_i\left(\sigma^*, \overline{\theta}_i\right)=1$, to the partition the set of σ^* and θ in which different dynamic properties can be observed.

To study the variation of the world inflation rate in the dynamic system of the core variables, we found a vector of the form $\left(\hat{\sigma}_i^*, \breve{\sigma}_i^*, \overline{\sigma}_i^*\right)$, such that each entry in this vector is one of three bifurcation values that partition the set of σ^* into four regions with defining characteristics. (a) $\sigma^* \in\left(-1, \hat{\sigma}_i^*\right)$, $\overline{\epsilon}_i\left(\sigma^*, \theta\right)<-1$ transpires, making unstable cyclical fluctuations observed along dynamic paths around the stationary core. (b) For $\sigma^* \in\left(\hat{\sigma}_i^*, \breve{\sigma}_i^*\right)$, $\overline{\epsilon}_i\left(\sigma^*, \theta\right) \in(-1,0)$ obtains, resulting in damped oscillations around the stationary. (c) $\overline{\epsilon}_i\left(\sigma^*, \theta\right) \in(0,1)$ occurs when $\sigma^* \in\left(\breve{\sigma}_i^*, \overline{\sigma}_i^*\right)$, causing stable monotonic dynamics around the steady-state core. (d) For values of the world inflation rate that are sufficiently high (i.e. when $\sigma^*>\overline{\sigma}_i^*$ holds), $\overline{\epsilon}_i\left(\sigma^*\right)>1$ ensues and dynamic paths diverge monotonically from the core steady-state.

Shifting our focus to backing of the money supply, for each variable indexed by i that belongs to the core system, there exists a vector $\left(\hat{\theta}_i, \breve{\theta}_i, \overline{\theta}_i\right)$ of bifurcation values of θ. We listed four mutually exclusive situations and discussed the range of the dynamic system given $\forall \theta \in[0,1]$. (a) If $\hat{\theta}_i<0<\overline{\theta}_i<1$, $\overline{\epsilon}_i\left(\sigma^*, \theta\right)>-1$ obtains, $\forall \theta \in[0,1]$, no unstable fluctuations are observed. Stable fluctuations arise, followed by monotonic stable dynamics and monotonic divergence, as θ increases. (b) If $0<\hat{\theta}_i<1<\overline{\theta}_i$; this rules out monotonic divergence. For

$\theta \in [0, \hat{\theta}_i)$, unstable fluctuations govern the dynamics around the stationary core. As θ increases, the oscillations gradually converge toward the steady-state core. Stable monotonic divergence ensues as θ continues to increase. (c) If $\hat{\theta}_i < 0 < 1 < \overline{\theta}_i$ obtains, there is no diverging dynamics. Damped oscillations are observed along dynamic paths, giving place to stable monotonic dynamics as θ increases. (d) If $0 < \hat{\theta}_i < \overline{\theta}_i < 1$, the full spectrum dynamics come about. Exploding oscillations take place for $\theta \in [0, \hat{\theta}_i)$, giving place to damped oscillations when $\theta \in [\hat{\theta}_i, \breve{\theta}_i)$. As θ continues to increase, stable monotonic dynamics ensue, followed by monotonic dynamics when $\theta > \overline{\theta}_i$.

The dynamics of $\overline{d}_{2,t+1}$ can be written as

$$\overline{d}_{2,t+1} = \Omega_0(\sigma^*) + \Omega_1(\sigma^*) \cdot \overline{\tau}_{t-1}, \tag{17}$$

Equation (17) is a first order linear difference equation in $\overline{\tau}_t$ which differs from the model economy of floating exchange rates. Then, the consumption by impatient households is given by $\lambda \cdot \overline{c}_{1,t} = f_1 - f_0 - (r_0^* - 1) \cdot f_0 + (r_0^* - 1) \cdot \overline{d}_{2,t+1}$. The dynamic properties of $\overline{c}_{1,t}$ are inherited from $\overline{d}_{2,t+1}$, indicating non-trivial dynamics. The eigenvalue $\Omega_1(\sigma^*)$ in the dynamics of $\overline{d}_{2,t+1}$ is a monotonically increasing function of the world inflation rate σ^*. We found that the dynamics of $\overline{d}_{2,t+1}$ follow unstable and non-cyclical fluctuations for values of σ^* sufficiently close to -1. As σ^* increases, the eigenvalue is inside the unit circle, with damped oscillations around the steady-state that turn gradually into stable monotonic dynamics when σ^* is sufficiently large. Similar dynamic properties of $(\overline{c}_{1,t}, \overline{c}_{2,t+1}) >> 0$ can be observed.

Similar to floating exchange rates, we illustrated the quantitative analysis with a numerical example in simulations following a reasonable set of parameters. Then, we reported the sensitive analysis of the policy parameters to the dynamic properties under fixed exchange rate regime. First, higher steady-state inflation promotes stable monotonic dynamics, eliminating instability and fluctuations altogether. Second, high reserve requirements increase the scope for endogenously-arising volatility. Finally, adding to the backing of the domestic money supply reduces the scope for unstable fluctuations while increasing the scope for stable monotonic dynamics[9].

In summary

Given the complexity of interaction between monetary policies and exchange rate regimes, we summarized the dynamic properties and instability in the model economy. First, all dynamic systems under a hard peg have first order difference equations, eliminating the possibility of cyclical fluctuations in the debt-structure vector that are typically associated with complex eigenvalues. Second, regarding the core dynamics, while a floating regime allows only for monotonic dynamics, full spectrum dynamics can be observed under a hard peg. We observed bifurcation of parameters which promotes endogenously-arising volatility around the stationary core under a fixed exchange rate regime. Regarding the foreign-debt structure and the state-contingent consumption, in some cases, fluctuations can be significantly large and explosive under floating exchange rates. A hard peg, instead, prevents explosive fluctuation in the first order difference equation systems.

[9] Simulation results are based on requests.

5. Conclusion

As the regulatory agencies and creditors learned from the historical mistakes, building the sophisticated macroeconomic policy and the rigorous financial regulations, they became aware that each financial crisis is unique and different. However, Diamond and Rajan (2009, p.606) note,

> "…This (the subprime crisis) is a crisis born in some ways of previous financial crises, which swept through the emerging markets in the late 1990s: East Asian economies collapsed, Russia defaulted, and Argentian, Brazil, and Turkey faced severe stress. A number of these countries became net exporters of financial capital…. Clearly, the net financial savings generated in one part of the world have to be absorbed by deficits elsewhere".

Some core features in the Asian crisis show important parallels to the 2007-2008 US financial crises. Private-sector over expansion activated the investment boom-bust cycle. Changes in investors' expectations led to the depreciation of currencies, bank runs, rapid foreign capital outflows, and dramatic economic downturns. This study investigates the monetary policy response in a model economy embedded with the stylized facts.

At a methodological level, this study provides a framework for analyzing interaction among monetary policies: fixed versus floating exchange rate regimes, domestic rate of money growth, regulations of multiple reserve requirements, and backing of the domestic money supply. We are fully aware that, in the broad scales of policy consideration in the aftermath of financial crises, the trade-offs of policy implementations are highly complex. Thus, this study compares policies from the standpoints of the dynamic properties, stability and volatility, and the scope of multiple equilibria. Under the same framework, the model can be extended to evaluate policies from steady-state welfare, fragility and volatility, and the scope of panic and non-panic equilibria under the Equilibrium Selection Mechanism (Wang and Hernandez 2011).

With respect to dynamic equilibria under floating exchange rates, oscillatory divergence is possible; in some cases, the fluctuation can be large and explosive. With respect to the properties of dynamic paths under a hard peg, dynamic systems are constructed by first order difference equations. Regarding the core, full spectrum dynamics can be observed under a fixed regime while a floating regime allowed only monotonic dynamics. There is a trade-off vis-à-vis the foreign-debt structure and state-contingent consumption: the floating exchange rate arrangement has second order difference equations that present either a saddle or source. A hard peg, instead, is only a small range for first order, simple oscillating dynamics.

Overall, the policy recommendations vary drastically across regimes: i) high and binding reserve requirements promote and extend the stability of dynamic equilibria under floating, while they increase endogenously arising volatility under fixed; ii) the backing of the money supply is a de-stabilizing policy parameter under floating, but it promotes stability under fixed; iii) the policy recommendations are exact opposites; floating requires high reserve requirements and a low backing of the money supply, but an economy with fixed exchange regime improves stability with a combination of low reserve requirements and a high backing of the money supply.

Table 1: Exchange Rate Regimes in the East-Asian Countries

Before the Crisis and After the Crisis

Country	Before/During the crisis	After the crisis
Japan	Free floating	Free floating
Philippines	Free floating	Free floating
China	Managed floating	Managed floating
Indonesia	Managed floating	Floating
Korea	Managed floating	Floating
Singapore	Managed floating	Managed floating
Thailand	Managed floating	Managed floating $\rightarrow$ floating
Malaysia	Managed floating	Fixed
Hong Kong	Fixed	Fixed

Source: Frankel et al (2002)

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Appendix A The Dynamic System under a Floating Exchange Rate

The general equilibrium system characterizes equilibrium variables. The domestic price level $\hat{p}_t$ clears the market for domestic real money balances:

$$\hat{M}_t / \hat{p}_t = \hat{z}_t = \phi_d \left(w + \hat{\tau}_t \right). \tag{A.1.1}$$

It leads to the equilibrium return of domestic real money balances

$$\hat{p}_t / \hat{p}_{t+1} = (1+\sigma)^{-1} \left(\hat{z}_{t+1} / \hat{z}_t \right) \tag{A.1.2}$$

and, using also the government budget constraint in equation (5), to the equilibrium laws of motion of z_t and τ_t, respectively.

$$\hat{z}_t = a_0(\sigma) + a_1(\sigma) \cdot \hat{z}_{t-1}, \tag{A.1.3}$$

$$\hat{\tau}_t = b_0(\sigma) + b_1(\sigma) \cdot \hat{z}_{t-1}, \tag{A.1.4}$$

where the reduced-form coefficients are given by

$$a_0 \equiv \frac{\phi_d \cdot w \cdot (1+\sigma)}{\left\{1 + \theta \cdot \phi_d + \sigma \cdot \left[1 - \phi_d \cdot (1-\theta)\right]\right\}}, a_1 \equiv \frac{\theta \cdot \phi_d \cdot \varepsilon \cdot (1+\sigma)}{\left\{1 + \theta \cdot \phi_d + \sigma \cdot \left[1 - \phi_d \cdot (1-\theta)\right]\right\}},\ b_0 \equiv \frac{a_0}{\phi_d} - w, b_1 \equiv \frac{a_1}{\phi_d}.$$

The representative bank's long-term investment in equilibrium follows

$$\hat{k}_{t+2} = k\left(\hat{z}_t\right) = c_0 + c_1 \cdot \hat{z}_t, \tag{A.1.5}$$

where $c_0 \equiv f_0 + \left(1 - \phi_d - \phi_f\right) \cdot \left(w + b_0\right), c_1 \equiv \left(1 - \phi_d - \phi_f\right) \cdot b_1$.

The market for foreign currency also clears when $\hat{q}_t \equiv \left(e_t \cdot \hat{Q}_t / \hat{p}_t\right) = \phi_f \cdot \left(w + \hat{\tau}_t\right) = \phi_f \cdot \hat{z}_t / \phi_d$. In equilibrium, q_t and b_t^* are governed by the following two reduced-form equations

$$\hat{q}_t = q\left(\hat{z}_t\right) = d_0 + d_1 \cdot \hat{z}_{t-1}, \tag{A.1.6}$$

$$\hat{b}_t = b\left(\hat{z}_t\right) = e_0 + e_1 \cdot \hat{z}_{t-1}, \tag{A.1.7}$$

where $d_0 \equiv \phi_f \cdot \left(w + b_0\right), d_1 \equiv \phi_f \cdot b_1$ and $e_0 \equiv \theta \cdot a_0, e_1 \equiv \theta \cdot a_1$.

Moreover, the endogenous growth rate of the supply of dollars in the domestic economy is given by

$$\left(\hat{Q}_{t+1}/\hat{Q}_t\right)=\left[\left(1+\sigma^*\right)\cdot\hat{z}_{t+1}\right]/\hat{z}_t\,, \tag{A.1.8}$$

while the nominal exchange rate follows:

$$\left(e_{t+1}/e_t\right)=\left[\left(1+\sigma\right)\cdot\hat{z}_t\right]/\left[\left(1+\sigma^*\right)\cdot\hat{z}_{t+1}\right]. \tag{A.1.9}$$

Finally, there are several conditions that characterize deposit contracts in equilibrium. One, the truth-telling condition, equation (2), holds. Two, the constraints on foreign credit must bind, and thus

$$\hat{d}_{0,t}+\hat{d}_{2,t+1}=f_0 \text{ and } \hat{d}_{1,t+1}+\hat{d}_{2,t+1}=f_1. \tag{A.1.10}$$

A.1. Stationary Equilibrium

The core dynamic system is de-coupled, inheriting its dynamics from $\hat{z}_t$. The stationary values of core variables are:

$$\begin{aligned}
\hat{z}&=\left\langle\phi_d\cdot w\cdot(1+\sigma)/\left(\sigma\cdot\left[1-\phi_d\cdot\left(1+\theta\cdot(\tilde{r}-1)\right)\right]+1-\phi_d\cdot\theta\cdot(\tilde{r}-1)\right)\right\rangle\\
\hat{\tau}&=\left\langle w\cdot\left\{\theta\cdot\phi_d\cdot(\tilde{r}-1)+\sigma\cdot\phi_d\cdot\left[\theta\cdot(\tilde{r}-1)+1\right]\right\}/\left(\sigma\cdot\left[1-\phi_d\cdot\left(1+\theta\cdot(\tilde{r}-1)\right)\right]+1-\phi_d\cdot\theta\cdot(\tilde{r}-1)\right)\right\rangle\\
\hat{q}&=\left[\phi_f\cdot w\cdot(1+\sigma)/\left(\sigma\cdot\left[1-\phi_d\cdot\left(1+\theta\cdot(\tilde{r}-1)\right)\right]+1-\phi_d\cdot\theta\cdot(\tilde{r}-1)\right)\right]\\
\hat{b}^*&=\left[\theta\cdot\phi_d\cdot w\cdot(1+\sigma)/\left(\sigma\cdot\left[1-\phi_d\cdot\left(1+\theta\cdot(\tilde{r}-1)\right)\right]+1-\phi_d\cdot\theta\cdot(\tilde{r}-1)\right)\right]\\
\hat{k}&=\xi_1(\sigma)+\xi_2(\sigma)\cdot\hat{z}
\end{aligned} \tag{A.1.11}$$

where $\xi_1(\sigma)\equiv f_0+\left[\left(1-\phi_d-\phi_f\right)\cdot w\cdot\left\{2+\theta\cdot\phi_d+\sigma\cdot\left[2-\phi_d\cdot(1-\theta)\right]\right\}\right]/\left\{1+\theta\cdot\phi_d+\sigma\cdot\left[1-\phi_d\cdot(1-\theta)\right]\right\}$ and $\xi_2(\sigma)\equiv\left[\left(1-\phi_d-\phi_f\right)\cdot\theta\cdot\tilde{r}\cdot(1+\sigma)\right]/\left\{1+\theta\cdot\phi_d+\sigma\cdot\left[1-\phi_d\cdot(1-\theta)\right]\right\}$. Notice that $\left(\hat{z},\hat{q},\hat{b}^*\right)$ are increasing in the policy parameters (σ,ϕ_d,θ) and that, as expected, $\hat{q}$ is increasing in ϕ_f. In addition, $\hat{\tau}$ is nonlinear in both σ and ϕ_d but monotonically increasing in θ. Finally, $\hat{k}$ is increasing in σ, but nonlinear in (ϕ_d, ϕ_f). The steady-state gross returns on domestic and foreign real money balances, the growth of the nominal exchange rate, and the growth rate of the real exchange rate, are all constant and equal to $(1+\sigma)^{-1}$, $(1+\sigma^*)$, $(1+\sigma)\cdot(1+\sigma^*)^{-1}$ and 1, respectively.

Based on the properties of the structure of foreign debt issued by domestic banks, we observed multiple stationary equilibria in this model economy with floating exchange rates. The general properties of the interior solution displayed by the stationary debt-structure in equilibrium depend on the different values that the policy parameter σ --the growth rate of the domestic money supply-- may take. The properties have to do with existence and the uniqueness of the equilibria. Thus, σ is a bifurcation parameter of the steady-state allocation given by $\left\{(\hat{z}, \hat{\tau}, \hat{q}, \hat{b}^*, \hat{k}), (\hat{d}_0, \hat{d}_1, \hat{d}_2), (\hat{c}_1, \hat{c}_2) \middle| \hat{l} = 0\right\}$, and so is the structure of the interest rates $(r_0^*, r_1^*, r_2^*) >> 1$. Note that the core in the steady-state $(\hat{z}, \hat{\tau}, \hat{q}, \hat{b}^*, \hat{k})$ is always unique and determinate since it is not associated with the vector $(r_0^*, r_1^*, r_2^* = R)$. However, for a fixed point in the parameter-space and for each stationary debt-structure vector $(\hat{d}_{0,j}, \hat{d}_{1,j}, \hat{d}_{2,j})$, there is typically a continuum of vectors of interest rates satisfying the equilibrium conditions.

A.2. Dynamics Equilibrium

A.2.1. The Core Dynamic Sub-system Under Floating

$$a_0 \equiv \frac{\phi_d \cdot w \cdot (1+\sigma)}{\left\{1+\theta\cdot\phi_d + \sigma\cdot\left[1-\phi_d\cdot(1-\theta)\right]\right\}}, a_1 \equiv \frac{\theta\cdot\phi_d\cdot\varepsilon\cdot(1+\sigma)}{\left\{1+\theta\cdot\phi_d + \sigma\cdot\left[1-\phi_d\cdot(1-\theta)\right]\right\}} \quad \text{(A.2.1)}$$

$$b_0 \equiv \frac{a_0}{\phi_d} - w, b_1 \equiv \frac{a_1}{\phi_d} \quad \text{(A.2.2)}$$

$$c_0 \equiv f_0 + \left(1-\phi_d-\phi_f\right)\cdot\left(w+b_0\right), c_1 \equiv \left(1-\phi_d-\phi_f\right)\cdot b_1 \quad \text{(A.2.3)}$$

$$d_0 \equiv \phi_f \cdot \left(w+b_0\right), d_1 \equiv \phi_f \cdot b_1 \tag{A.2.4}$$

$$e_0 \equiv \theta \cdot a_0, e_1 \equiv \theta \cdot a_1 \tag{A.2.5}$$

A.2.2. The Foreign-Debt Sub-system Under Floating

$$g_0 \equiv \frac{\lambda \cdot R \cdot \left(1-\phi_d-\phi_f\right)\cdot\left(w+b_0\right)}{r_1^* \cdot \left(r_0^*-1\right)} + \frac{\lambda \cdot \phi_f \cdot \left(w+b_0\right)}{r_1^* \cdot \left(r_0^*-1\right)\cdot\left(1+\sigma^*\right)} + \frac{\lambda \cdot \phi_d \cdot \left(w+b_0\right)\cdot a_1}{r_1^* \cdot \left(r_0^*-1\right)\cdot(1+\sigma)} - \frac{f_1 - r_0^* \cdot f_0}{\left(r_0^*-1\right)} \tag{A.2.6}$$

$$g_1 \equiv \frac{\lambda \cdot R \cdot \left(1-\phi_d-\phi_f\right)\cdot b_1}{r_1^* \cdot \left(r_0^*-1\right)} + \frac{\lambda \phi_f \cdot b_1}{r_1^* \cdot \left(r_0^*-1\right)\cdot\left(1+\sigma^*\right)} + \frac{\lambda \cdot \left(a_1\right)^2}{r_1^* \cdot \left(r_0^*-1\right)\cdot(1+\sigma)} > 0 \tag{A.2.7}$$

$$g_2 \equiv \left(\lambda \cdot a_0 \cdot a_1\right) \Big/ \left[r_1^* \cdot \left(r_0^*-1\right)\cdot(1+\sigma)\right] > 0 \tag{A.2.8}$$

$$g_3 \equiv \left[\lambda \cdot \left(a_0\right)^2\right] \Big/ \left[r_1^* \cdot \left(r_0^*-1\right)\cdot(1+\sigma)\right] > 0 \tag{A.2.9}$$

A.2.3. The State-Contingent Consumption Sub-system Under Floating

$$h_0 \equiv \left(f_1 - r_0^* \cdot f_0\right) \Big/ \lambda, h_1 \equiv \left(r_0^*-1\right) \Big/ \lambda > 0 \tag{A.2.10}$$

$$j_0 \equiv \frac{R \cdot c_0}{(1-\lambda)} - \frac{r_1^* \cdot f_1}{(1-\lambda)} + \frac{\phi_f \cdot \left(w+b_0\right)}{(1-\lambda)\cdot\left(1+\sigma^*\right)} + \frac{\phi_d \cdot \left(w+b_0\right)\cdot a_1}{(1-\lambda)\cdot(1+\sigma)} \tag{A.2.11}$$

$$j_1 \equiv -\left(R - r_1^*\right) \Big/ (1-\lambda) < 0 \tag{A.2.12}$$

$$j_2 \equiv \frac{R \cdot \left(1-\phi_d-\phi_f\right)\cdot b_1}{(1-\lambda)} + \frac{\phi_f \cdot b_1}{(1-\lambda)\cdot\left(1+\sigma^*\right)} + \frac{\left(a_1\right)^2}{(1-\lambda)\cdot(1+\sigma)} > 0 \tag{A.2.13}$$

$$j_3 \equiv \left(a_0 \cdot a_1\right) \Big/ \left[(1-\lambda)\cdot(1+\sigma)\right] > 0 \tag{A.2.14}$$

$$j_4 \equiv \left(a_0\right)^2 \Big/ \left[(1-\lambda)\cdot(1+\sigma)\right] > 0 \tag{A.2.15}$$

$$n_0 \equiv j_0 + j_1 \cdot g_0 \tag{A.2.16}$$

$$n_1 \equiv j_2 + j_1 \cdot g_1 \tag{A.2.17}$$

$$n_2 \equiv j_3 + j_1 \cdot g_2 \tag{A.2.18}$$

$$n_3 \equiv j_4 + j_1 \cdot g_3 \tag{A.2.19}$$

1. The Dynamic Sub-system of the Debt-Structure Under Floating

We focused on the dynamics properties of $\hat{d}_{2,t+1}$, which is in turn inherited by $\hat{d}_{0,t}$ and by $\hat{d}_{1,t+1}$.

1.1 Part 1 of De-coupled Sub-system

To solve for the dynamic path, we augmented the state-space by using $\hat{y}_{t+1} = \hat{z}_t$, which reduces the order of the system to a first order nonlinear dynamic system in $(\hat{z}_t, \hat{y}_t)$ as shown below:

$$\begin{aligned} \hat{d}_{2,t+1} &= g_0 + g_1 \cdot \hat{y}_t + g_2 \cdot \left(\frac{\hat{y}_t}{\hat{z}_t}\right) + g_3 \cdot \left(\frac{1}{\hat{z}_t}\right) \\ \hat{y}_{t+1} &= \hat{z}_t \end{aligned} \tag{A.2.20}$$

We linearized the system (A.2.20) in a neighborhood of a steady-state. The Jacobian matrix of the linearized system is $J(\hat{z},\hat{y}) = \begin{bmatrix} J_{11} & J_{12} \\ 1 & 0 \end{bmatrix}$, where $J_{11} \equiv \left.\frac{\partial \hat{d}_{2,t+1}}{\partial \hat{z}_t}\right|_{(\hat{z},\hat{y})} = T(J)$ is the trace of the matrix and $J_{12} \equiv \left.\frac{\partial \hat{d}_{2,t+1}}{\partial \hat{y}_t}\right|_{(\hat{z},\hat{y})} = -D(J)$ is the negative of the determinant. Both trace and determinant are continuous monotonic functions of the parameters and are given by the expressions:

$$Tr(J) = -\left(\frac{g_2}{\hat{z}} + \frac{g_3}{\hat{z}^2}\right) < 0 \Rightarrow \lambda_1 + \lambda_2 < 0, \tag{A.2.21}$$

$$Det(J) = -\left(g_1 + \frac{g_2}{\hat{z}}\right) < 0 \Rightarrow \lambda_1 \cdot \lambda_2 < 0 . \tag{A.2.22}$$

It is apparent from (A.2.21-22), that the pair of eigenvalues has opposite signs and that the negative root dominates in magnitude, indicating non monotonic dynamics.

1.2 Part 2 of De-coupled Sub-system

$$\hat{d}_{0,t} = \left(f_0 - g_0\right) - g_1 \hat{y}_t - g_2 \left(\frac{\hat{y}_t}{\hat{z}_t}\right) - g_3 \left(\frac{1}{\hat{z}_t}\right)$$
$$\hat{y}_{t+1} = \hat{z}_t$$

$$J(\hat{z},\hat{y}) = \begin{bmatrix} T & -D \\ 1 & 0 \end{bmatrix},$$

$$Tr\left(J\right) = \left(\frac{g_2}{\hat{z}} + \frac{g_3}{\hat{z}^2}\right) > 0 \Rightarrow \lambda_1 + \lambda_2 > 0$$

$$Det\left(J\right) = \left(g_1 + \frac{g_2}{\hat{z}}\right) > 0 \Rightarrow \lambda_1 \cdot \lambda_2 > 0$$

1.3 Part 3 of De-coupled Sub-system

$$\hat{d}_{1,t+1} = \left(f_1 - g_0\right) - g_1 \hat{y}_t - g_2 \left(\frac{\hat{y}_t}{\hat{z}_t}\right) - g_3 \left(\frac{1}{\hat{z}_t}\right)$$
$$\hat{y}_{t+1} = \hat{z}_t$$

$$J(\hat{z},\hat{y}) = \begin{bmatrix} T & -D \\ 1 & 0 \end{bmatrix},$$

$$Tr\left(J\right) = \left(\frac{g_2}{\hat{z}} + \frac{g_3}{\hat{z}^2}\right) > 0 \Rightarrow \lambda_1 + \lambda_2 > 0$$

$$Det\left(J\right) = \left(g_1 + \frac{g_2}{\hat{z}}\right) > 0 \Rightarrow \lambda_1 \cdot \lambda_2 > 0$$

Obviously, 1.2 and 1.3 have identical dynamics.

2. The Dynamic Sub-system of the State-Contingent Consumption Under Floating

The state-contingent consumption vector displays non trivial dynamics, as seen in (13a-b). Equation (13a) is a second order, nonlinear, difference equation in $\hat{z}_t$.

2.1 Part 1 of the Sub-system: Consumption by Impatient Agents

Similar to the dynamic system of debt structure, we reduced the order of the system to a first order nonlinear dynamic system in $\left(\hat{z}_t, \hat{y}_t\right)$ as shown below:

$$\begin{aligned} \hat{c}_{1,t} &= \left(h_0 + h_1 \cdot g_0\right) + h_1 \cdot g_1 \cdot \hat{y}_t + h_1 \cdot g_2 \cdot \left(\frac{\hat{y}_t}{\hat{z}_t}\right) + h_1 \cdot g_3 \cdot \left(\frac{1}{\hat{z}_t}\right) \\ \hat{y}_{t+1} &= \hat{z}_t \end{aligned} \qquad \text{(A.2.23)}$$

Both trace and determinant are continuous monotonic functions of the parameters, and given by the expressions:

$$Tr\left(J\right) = -h_1 \cdot \left(\frac{g_2}{\hat{z}} + \frac{g_3}{z^2}\right) < 0 \Rightarrow \lambda_1 + \lambda_2 < 0 , \qquad \text{(A.2.24)}$$

$$Det\left(J\right) = -h_1 \cdot \left(g_1 + \frac{g_2}{\hat{z}}\right) < 0 \Rightarrow \lambda_1 \cdot \lambda_2 < 0 . \qquad \text{(A.2.25)}$$

It is apparent from (A.2.24-25), that the pair of eigenvalues has opposite signs and that the negative root dominates in magnitude, indicating non monotonic dynamics.

2.2 Part 2 the Sub-system: Consumption by Patient Agents

$$\begin{aligned} \hat{c}_{2,t+1} &= n_0 + n_1 \cdot \hat{y}_t + n_2 \cdot \left(\frac{\hat{y}_t}{\hat{z}_t}\right) + n_3 \cdot \left(\frac{1}{\hat{z}_t}\right) \\ \hat{y}_{t+1} &= \hat{z}_t \end{aligned} \qquad \text{(A.2.26)}$$

Both trace and determinant are continuous monotonic functions of the parameters, and given by the expressions:

$$Tr(J) = -\left(\frac{n_2}{\hat{z}} + \frac{n_3}{\hat{z}^2}\right), \tag{A.2.27}$$

$$Det(J) = -\left(n_1 + \frac{n_2}{\hat{z}}\right). \tag{A.2.28}$$

The pair of eigenvalues has opposite signs, and the negative root dominates in magnitude, indicating non monotonic dynamics.

Appendix B The Dynamic System under a Fixed Exchange Rate

The equilibrium laws of motion in equations (A.1.3) must be modified, and the following two equations obtain:

$$\overline{\tau}_t = \eta_1\left(\sigma^*\right) + \eta_2\left(\sigma^*\right)\cdot\overline{\tau}_{t-1}, \tag{B.1.1}$$

$$\overline{z}_t = \rho_1\left(\sigma^*\right) + \rho_2\left(\sigma^*\right)\cdot\overline{\tau}_{t-1}, \tag{B.1.2}$$

where the coefficients are $\eta_1\left(\sigma^*\right) \equiv \left\langle \phi_d \cdot w \cdot \left\{\left(1+\sigma^*\right)\cdot\left[\theta\cdot\left(\tilde{r}-1\right)+1\right]-1\right\} \big/ И\left(\sigma^*\right)\right\rangle$,

$\eta_2\left(\sigma^*\right) \equiv \left\{\phi_d \cdot \left[\theta\cdot\tilde{r}\cdot\left(1+\sigma^*\right)-1\right] \big/ И\left(\sigma^*\right)\right\}$, $\rho_2\left(\sigma^*\right) \equiv \phi_d \cdot \eta_2\left(\sigma^*\right)$, $\rho_1\left(\sigma^*\right) \equiv \phi_d \cdot \left[w + \eta_1\left(\sigma^*\right)\right]$, and

$И\left(\sigma^*\right) \equiv \left(1+\sigma^*\right)\cdot\left[1-\phi_d\cdot\left(1-\theta\right)\right]$. Notice that equations above are first order linear difference equations in τ_t. Under this hard peg, the dynamics of the system originates in τ_t instead of z_t, as it was the case under floating exchange rates. We obtained the equilibrium laws of motion in a hard peg:

$$\overline{q}_t = \chi_1\left(\sigma^*\right) + \chi_2\left(\sigma^*\right)\cdot\overline{\tau}_{t-1}, \tag{B.1.3}$$

$$\overline{b}_t^* = \psi_1\left(\sigma^*\right) + \psi_2\left(\sigma^*\right)\cdot\overline{\tau}_{t-1}, \tag{B.1.4}$$

where $\chi_1(\sigma^*) \equiv \phi_f \cdot \rho_1(\sigma^*) / \phi_d$, $\chi_2(\sigma^*) \equiv \phi_f \cdot \rho_2(\sigma^*) / \phi_d$, $\psi_1(\sigma^*) \equiv \theta \cdot \rho_1(\sigma^*)$ and $\psi_2(\sigma^*) \equiv \theta \cdot \rho_2(\sigma^*)$. Next, the nominal exchange rate becomes

$$(\overline{e}_{t+1}/\overline{e}_t) = (e/e) = 1. \tag{B.1.5}$$

The equilibrium conditions relate to the deposit contract offered by banks are as follows. One, the truth-telling constraint in (2) holds. Two, the constraints on foreign credit in (6) and (7) continue to bind. Three, the equilibrium law of motion for the long-term investment is now given by

$$\overline{k}_{t+1} = \varsigma_1(\sigma^*) + \varsigma_2(\sigma^*) \cdot \overline{\tau}_{t-1}, \tag{B.1.6}$$

where $\varsigma_1(\sigma^*) \equiv f_0 + (1-\phi_d-\phi_f) \cdot \rho_1(\sigma^*) / \phi_d$ and $\varsigma_2(\sigma^*) \equiv (1-\phi_d-\phi_f) \cdot \rho_2(\sigma^*) / \phi_d$. Four, the total return on domestic and foreign currency reserves under this policy regime are given, respectively, by the following two equations:

$$\phi_d \cdot (\overline{p}_t/\overline{p}_{t+1}) \cdot (w+\overline{\tau}_t) = \mu_1(\sigma^*) + \mu_2(\sigma^*) \cdot \overline{\tau}_{t-1}, \tag{B.1.7}$$

$$\phi_f \cdot (p_t^*/p_{t+1}^*) \cdot (w+\overline{\tau}_t) = \upsilon_1(\sigma^*) + \upsilon_2(\sigma^*) \cdot \overline{\tau}_{t-1}, \tag{B.1.8}$$

where the coefficients are $\mu_1(\sigma^*) \equiv \rho_1(\sigma^*)/(1+\sigma^*)$, $\mu_2(\sigma^*) \equiv \rho_2(\sigma^*)/(1+\sigma^*)$, $\upsilon_1(\sigma^*) \equiv \chi_1(\sigma^*)/(1+\sigma^*)$ and $\upsilon_2(\sigma^*) \equiv \chi_2(\sigma^*)/(1+\sigma^*)$. Five, the space-contingent commodities are governed by

$$\lambda \cdot \overline{c}_{1,t,j} = f_1 - r_0^* \cdot f_0 + (r_0^*-1) \cdot \overline{d}_{2,t+1,j} \tag{B.1.9}$$

$$(1-\lambda) \cdot \overline{c}_{2,t+1,j} = \omega_1(\sigma^*) + \omega_2(\sigma^*) \cdot \overline{\tau}_{t-1} - r_1^* \cdot f_1 - (r_2^* - r_1^*) \cdot \overline{d}_{2,t+1,j}, \tag{B.1.10}$$

where the parameters are $\omega_1(\sigma^*) \equiv r_2^* \cdot \varsigma_1(\sigma^*) + \mu_1(\sigma^*) + \upsilon_1(\sigma^*)$ and $\omega_2(\sigma^*) \equiv r_2^* \cdot \varsigma_2(\sigma^*) + \mu_2(\sigma^*) + \upsilon_2(\sigma^*)$.

Stationary Equilibria

The five variables that belong to the core, $\left(\overline{\tau}_t, \overline{z}_t, \overline{q}_t, \overline{b}_t^*, \overline{k}_{t+1}\right)$, are determinate under a fixed exchange rate regime whenever an equilibrium exists since they do not depend on the foreign interest rates $\left(r_0^*, r_1^*, r_2^*\right)$. We obtain the steady-state values for the variables in the core in the following five expressions:

$$\begin{aligned}
\overline{\tau} &= \eta_1\left(\sigma^*\right)/\left[1-\eta_2\left(\sigma^*\right)\right] = \left\langle \phi_d \cdot w \cdot \left\{\left(1+\sigma^*\right)\cdot\left[1+\theta\cdot\left(\tilde{r}-1\right)\right]-1\right\}/\mathrm{M}\left(\sigma^*\right)\right\rangle \\
\overline{z} &= \left[\phi_d \cdot w \cdot \left(1+\sigma^*\right)/\mathrm{M}\left(\sigma^*\right)\right] \\
\overline{q} &= \left[\phi_f \cdot w \cdot \left(1+\sigma^*\right)/\mathrm{M}\left(\sigma^*\right)\right] \\
\overline{b^*} &= \left[\theta \cdot \phi_d \cdot w \cdot \left(1+\sigma^*\right)/\mathrm{M}\left(\sigma^*\right)\right] \\
\overline{k} &= f_0 + \left(1-\phi_d-\phi_f\right)\cdot w + \left\langle \phi_d \cdot \left(1-\phi_d-\phi_f\right)\cdot\left\{\left(1+\sigma^*\right)\cdot\left[\theta\cdot\left(\tilde{r}-1\right)+1\right]-1\right\}/\mathrm{M}\left(\sigma^*\right)\right\rangle
\end{aligned} \quad ,\text{(B.1.11)}$$

Here, $\mathrm{M}\left(\sigma^*\right)\equiv\left(1+\sigma^*\right)-\phi_d\cdot\left\{\left(1+\sigma^*\right)\cdot\left[1+\theta\cdot\left(\tilde{r}-1\right)\right]-1\right\}$, where $\left(1+\sigma^*\right)\cdot\left[1+\theta\cdot\left(\tilde{r}-1\right)\right]>1$ holds, $\forall\sigma^*>-1$. Given the latter, we found it reasonable to restrict our attention to allocations where $\left(1+\sigma^*\right)>\phi_d\cdot\left\{\left(1+\sigma^*\right)\cdot\left[1+\theta\cdot\left(\tilde{r}-1\right)\right]-1\right\}>0$ holds, $\forall\sigma^*>-1$. It follows that $\overline{\tau}>0$, and $\left(\partial\overline{\tau}/\partial\sigma^*\right)>0$ obtains.

The foreign-debt-structure vector $\left(\overline{d}_0, \overline{d}_1, \overline{d}_2\right) = \left(f_0-\overline{d}_2, f_1-\overline{d}_2, \overline{d}_2\right) >> 0$. The foreign long-term debt in a stationary equilibrium with a hard peg is given by

$$\overline{d_2} = \Omega_0\left(\sigma^*\right) + \Omega_1\left(\sigma^*\right)\cdot\overline{\tau}\,, \quad \text{(B.1.12)}$$

where $\Omega_0\left(\sigma^*\right)\equiv\left\{\lambda\cdot\left(1+\sigma^*\right)r_2^*\cdot\varsigma_1\left(\sigma^*\right)+\lambda\cdot\left(\phi_d+\phi_f\right)\left[\eta_1\left(\sigma^*\right)-w\right]-r_1^*\cdot\left(1+\sigma^*\right)\cdot\left(r_0^*-r_1^*\right)\right\}/\left[r_1^*\cdot\left(1+\sigma^*\right)\cdot\left(r_0^*-r_1^*\right)\right]$, and $\Omega_1\left(\sigma^*\right)\equiv\lambda\cdot\left[\left(1+\sigma^*\right)\cdot r_2^*\cdot\varsigma_2\left(\sigma^*\right)+\left(\phi_d+\phi_f\right)\cdot\eta_2\left(\sigma^*\right)\right]/\left[r_1^*\cdot\left(1+\sigma^*\right)\cdot\left(r_0^*-r_1^*\right)\right]$. The vector of state-contingent consumption and the expected utility obtain from

$$\lambda\cdot\overline{c}_1 = f_1 - r_0^*\cdot f_0 + \left(r_0^*-1\right)\cdot\Omega_0\left(\sigma^*\right) + \left(r_0^*-1\right)\cdot\Omega_1\left(\sigma^*\right)\cdot\overline{\tau}\,, \quad \text{(B.1.13)}$$

$$\left(1-\lambda\right)\cdot\overline{c}_2 = \Sigma_0\left(\sigma^*\right) - \left(R-r_1^*\right)\cdot\Omega_0\left(\sigma^*\right) + \left[\Sigma_1\left(\sigma^*\right)-\left(R-r_1^*\right)\cdot\Omega_1\left(\sigma^*\right)\right]\cdot\overline{\tau}\,, \quad \text{(B.1.14)}$$

$$\overline{U} = \lambda\cdot\ln\left(\overline{c}_1\right) + \left(1-\lambda\right)\cdot\ln\left(\overline{c}_2\right). \quad \text{(B.1.15)}$$