A Phase Transactions of the USD/JPY Exchange Rate

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Abstract.

In this paper, the USD/JPY exchange rate is modeled by a non-linear differential equation model, which consists of a trend noise only. The most significant finding is that, based on this model, conflicts of currency policies between the US and Japan occurs whenever big errors between the model and a real exchange rate emerges. The latest error is caused by the BoJ's ultra-losing monetary policy, which leads to strong depreciation of the USD/JPY. Additionally, I discuss why the real exchange rate is staying difficult on the around equilibrium exchange rate, caused by even a simple noise.

Keywords: Real Exchange Rate (RER), Future Exchange Rate (FER), Monetary Policy, Non-linear Differential Equation (NDE), Noise Analysis

JEL classification: C32, C61, E52, F31, G02

Even Adam Smith, the Canny Scot whose monumental book, "The Wealth of Nations'(1776), represents the beginning of modern economics or political economy-even he was so thrilled by the recognition of an order in the economic system that he proclaimed the mystical principle of the "invisible hand": that each individual in pursuing his own selfish good was led, as if by an invisible hand, to achieve the best good of all, so that any interference with free competition by government was almost certain to be injurious. --Paul Samuelson (1948).

1. Introduction

Many models estimate the equilibrium exchange rate (EER)[1] and future exchange rate (FER). The two main methods of modeling the exchange rate are via macroeconomic fundamentals and random walk (RW). Macroeconomic fundamentals are described by money supplies, outputs, inflation, interest rates, etc. Meese and Rogoff (1983a, b) use macroeconomic fundamentals and traditional economic theory to model the FER. Yoshimori[2] (2003) suggests that the EER and the FER can be described by macroeconomic fundamentals. Nelson (1995) and Chinn and Meese (1995) conclude that the FER can be described by a RW model, seeing that the FER has misaligned widely against the price which is predicted by macroeconomic fundamentals. Bellgard (1999) also discusses the FER based on the RW hypothesis. Kilian (2001) concludes that the RW is more likely to remain elusive for the foreseeable future, and Rossi (2006) rejects the hypothesis of the RW on the exchange rates. Additionally, Ca' Zorzi (2013) shows that a half-life Purchasing Power Parity (PPP) model can forecast the FER, rather than the RW model,

[1] "A foreign exchange rate is the price of one currency in terms of another, which is typically set in financial markets by supply and demand for currencies." McCown, Pollard and Weeks (2007). "Supply and demand for currencies are strongly influenced by conditions in the market for other financial assets, as well as market perceptions of underlying real macroeconomics conditions. To be described the equilibrium exchange rate (EER) with long-term, there are of course a lot of approaches with econometric models based on fundamentals." The approaches can be classified into three main types: 1) Purchasing Power Parity (PPP), 2) Behavioral Equilibrium Exchange Rate (BEER), 3) Fundamental Equilibrium Exchange Rate (FEER). PPP was first introduced by Cassel (1916 and 1918). The definition of the exchange rate is a ratio of home and foreign prices. The BEER approach, modeled by Clark and MacDonald (1998), provides a real exchange rate and an explanation for the slow reversion to the PPP. In the BEER approach, the exchange rate is determined as an actual value of economic fundamentals. The FEER, formalized by Williamson (1983 and 1994), is defined as adjusting the country's internal and external balance. It ignores the short-run cycle and concentrates on components that persist in the medium-term. Obstfeld and Rogoff (2005) and Blanchard, Giavazzi, and Sa (2005) state the notion of the EER with current account changes figured prominently on global imbalance.

[2] The model is a state-of-art econometric model with a dynamical system based on integrating fractal theory with traditional econometric methods; 1) by a measurement of a fractal dimension - how many factors should be described the USD/JPY exchange rate and 2) by two principle components - the exchange rate is described.

both short-term and long-term. Notwithstanding, there are still not any models which have completely succeeded to model the EER and FER.

This paper uses a non-linear differential equation (NDE) model based on the SY-RDS[3] (Shikata and Yoshimori- Rally Dynamic System (See Yoshimori (2017a)) to model the real exchange rate (RER). The model uses a unique conceptualization. For each time interval, I derive a linear differential equation (LDE) from the time series of the exchange rate, whose solution approximates the data, including differentials up to 2^{nd} and 3^{rd} order. The NDE is better than the SY-RDS, diminishing errors between the NDE model and RER drastically. The NDE consists of combinations of the 2^{nd}- and 3^{rd}-order differential, through a low-pass filter. The filter removes simple noise (a white noise: randomness). In fact, the time series is influenced by noise which is not a white noise but a trend noise (i.e. a pink noise) - a consecutive price actions such as appreciation or depreciation.

The first discussion is an ultra-loose monetary policy by Bank of Japan (BoJ): Quantitative and Qualitative Monetary Easing (QQE), which is derived from Abenomics, compared with the monetary policy in the late 1990s to early 2000s[4], induces the big error, $\frac{d^3}{dt^3}\left(f(t)\right)$ in a stage of an "input" and the real exchange rate (RER). The implication of the error is that the monetary policy after Feb. 2014[5] including QQE2 leads to an accelerating depreciation of the USD/JPY. This may be a strategy of the Japanese government, namely, the Japanese economy is boosted by recovering profits from the export sector.

[3] The unique point is that the differential equation is expressed by both market atmosphere (investors' psychology) and control system by government – for instance, intervention, monetary policy, or political accord."
[4] See Ito (2005)
[5] https://www.boj.or.jp/en/announcements/release_2014/k140218a.pdf

The second discussion is why the RER ignores the EER, which is estimated in the medium- and long-term by economic theories. More faculty, the RER cannot hover around the EER walks unsteady and trajects speedy around the EER- for instance, the RER has misalignment against the EER- the Purchasing Power Parity (PPP). What causes the RER misalignment? The reason is that the EER is unable to update quickly against updated information. Even so, why can't the market restore the EER quickly? Specifically, why do investors often forget or ignore the value of "fundamentals"? The answer is that the RER is weakly coiled around the EER so that the entropy of the RER around the EER is enlarged by a white noise, which is produced by the errors between the NDE model and RER. More concretely, walking the RER like the RW is caused by the white noise – psychology from technical chart analysis, a statement of central banks or executives, news headlines, a geopolitical risk or an option price – and leads to a shifted price range. Accordingly, it would be difficult for the RER to hover an equilibrium point such as the EER.

The remainder of the paper is organized as follows. Section 2 describes data and calibration. Section 3 explains the methodology and empirical results. In Section 4, I present an implication from an empirical result: an indemnification of an expansionary monetary policy (4-1) and allowing difficulty to movement along the equilibrium exchange rate (4-2). The last section contains concluding remarks.

2. Data and Calibration

My empirical analysis is based on the monthly USD/JPY exchange rate time series, obtained from Bloomberg (see Fig. 1). The data x_t were sampled monthly (from October 1985 to May 2017). I selected the end date to coincide with the Plaza Accord on September 22, 1985, which embodied a new regime, although the actual shifting from fixed to floating exchange rates occurred with the Smithsonian Agreement in 1973 after the Bretton Woods Agreements in 1971.

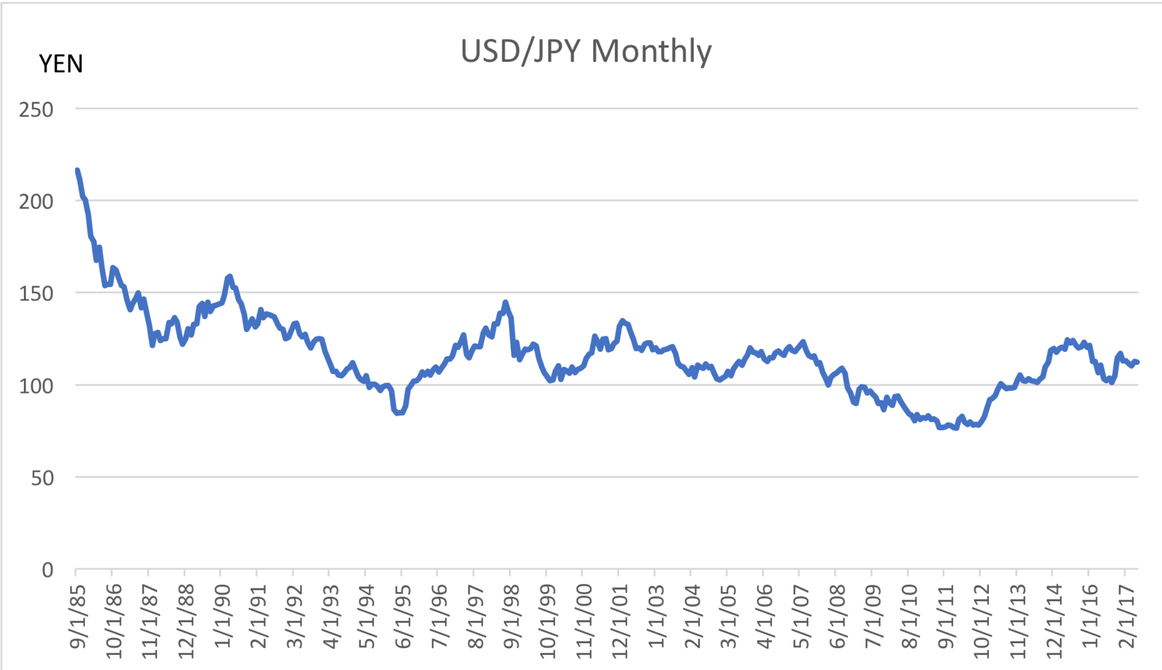

Fig.1

3. Methodology and Empirical Results

3-1 Methodology

The time series of the USD/JPY exchange rate isn't formed simply by a white noise (a randomness), but a pink noise (a trend noise) (See Yoshimori 2017a). Factually, the time series is formed by the trends, which are consecutive of appreciation or depreciation caused by a pink noise, so the pink noise has a better fitting than the white noise. The model is based on the standard theory of differential equations, through the SY-RDS model with a linear differential equation (Yoshimori and Shikata (2017a, b)) [6].

By combinations of the 2^{nd}- and 3^{rd}- order differential, the errors between a non-linear differential equation (NDE) model and a real exchange rate (RER) can be decreased remarkably compared with the SY-RDS model. I show the following method to construct the NDE.

Let $f(t)$ denote the monthly USD/JPY exchange rates. Given the exchange rate $f(t)$, we take the partial derivatives $\frac{d}{dt}(f(t))$, $\frac{d^2}{dt^2}(f(t))$, $\frac{d^3}{dt^3}(f(t))$, etc. as additional data sets to perform factor analysis of the exchange rate.

For given data sets $\frac{d^2}{dt^2}(f(t))$, $\frac{d^3}{dt^3}(f(t))$, and $f(t)$, the core part of the factor analysis is to express $f(t)$ as a nonlinear combination of $\frac{d^2}{dt^2}(f(t))$, $\frac{d^3}{dt^3}(f(t))$ that is, to write $f(t)$ as

$$f(t) = a\left(\frac{d^3}{dt^3}(f(t))\right)^2 + b\frac{d^2}{dt^2}(f(t)) \cdot \frac{d^3}{dt^3}(f(t)) \qquad \text{3-1}$$

[6] The ordinary approach to differential equations is as follows:

1) View the model as a network on a set of parts, whose characters are assumed to be well described by simple functions.
2) Describe the model equalizing the simple functions, corresponding to the network. The connections of the parts are expressed by the equalities.
3) This equalization gives equations, called differential equations, because they usually include 1^{st}-, 2^{nd}- or, 3^{rd}- order differentials in the time variable.
4) Solve or simulate these differential equations, to obtain mainly a periodic or converging solution.
5) If several models are discussed, the obtained solutions are compared with real data to choose the most-appropriate model.

This approach may include logical difficulty concerning the uniqueness of the derived model. In physics, more precisely in mechanics and electrostatics, Newton's law and Maxwell's law strictly describe the role of the differentials. However, in economics there is no notion of force, and no mathematical terminology to express (economic) atmosphere. This leads to ambiguity in establishing an economical model for given real data.

allowing for error terms. In practice, the coefficients a and b are chosen to minimize the L^2 norm (see Appendix A) of error terms, i.e. the difference $\left[f(t) - \left\{a\left[\frac{d^3}{dt^3}\left(f(t)\right)\right]^2 + b\,\frac{d^2}{dt^2}\left(f(t)\right) \cdot \frac{d^3}{dt^3}\left(f(t)\right)\right\}\right]$ over the time interval under consideration.

In finding a and b, I consider up to the third derivative $\frac{d^3}{dt^3}\left(f(t)\right)$. The shifted data is then smoothed by suitable moving averages of $\frac{d^3}{dt^3}\left(f(t)\right)$. The moving average of $\frac{d^3}{dt^3}\left(f(t)\right)$ is a data set $MA(t)$ obtained from $\int_t^{t+a} \frac{d^3}{dt^3}\left(f(t)\right)$, which reduces the power $\sqrt{a^2(n) + b^2(n)}$ of the trigonometric function $a(n)\sin(nt) + b(n)\cos(nt)$ to $\frac{1}{n}$, that is, the power of the moving averaged trigonometric function $a(n)\sin(nt) + b(n)\cos(nt)$ is less than $\frac{1}{n}\sqrt{a^2(n) + b^2(n)} = \frac{original\ power}{n}$. In this sense, the moving average is called the low-pass filter in electronics engineering.

Since the validity of the equation depends on the error, additionally, I expect that the error function describes the structure of reality by a suitable interpretation of the terms in the equation[7]. Regarding the general solution of NDE, I show the Appendix B.

3-2. Empirical Result

[7] My insight is that psychological phenomenon can be described by differential equations, and my definitions are below. Investors' psychology (i.e. minds) are captured by the 1st-order differential. The up-and-down trend of market atmosphere created by investors' psychology is captured by the 2nd-order differential. The control power of the market atmosphere is captured by the 3rd-order differential. A more detailed definition of market atmosphere is the following: Investors think that today's price action is influenced by yesterday's price action. If the US dollar appreciates today, for example, investors anticipate that tomorrow's price will also appreciate.

From the data, I deduced the following differential equation for the smoothed monthly time series of the USD/YEN exchange rate:

$$f(t) = \left(\frac{d^3}{dt^3}\big(f(t)\big)\right)^2 + 2\frac{d^2}{dt^2}\big(f(t)\big)\cdot\frac{d^3}{dt^3}\big(f(t)\big) \qquad 3\text{-}2$$

To justify that the above is optimized, I compare the L^2 norm of $\frac{d^2}{dt^2}\big(f(t)\big), \frac{d^3}{dt^3}\big(f(t)\big)$; the former is 0.03 while the latter is 0.10.

Fig.2 shows the smoothed square, and the graph shows that the monthly USD/JPY exchange rate, is controlled within the indicated range of the equation.

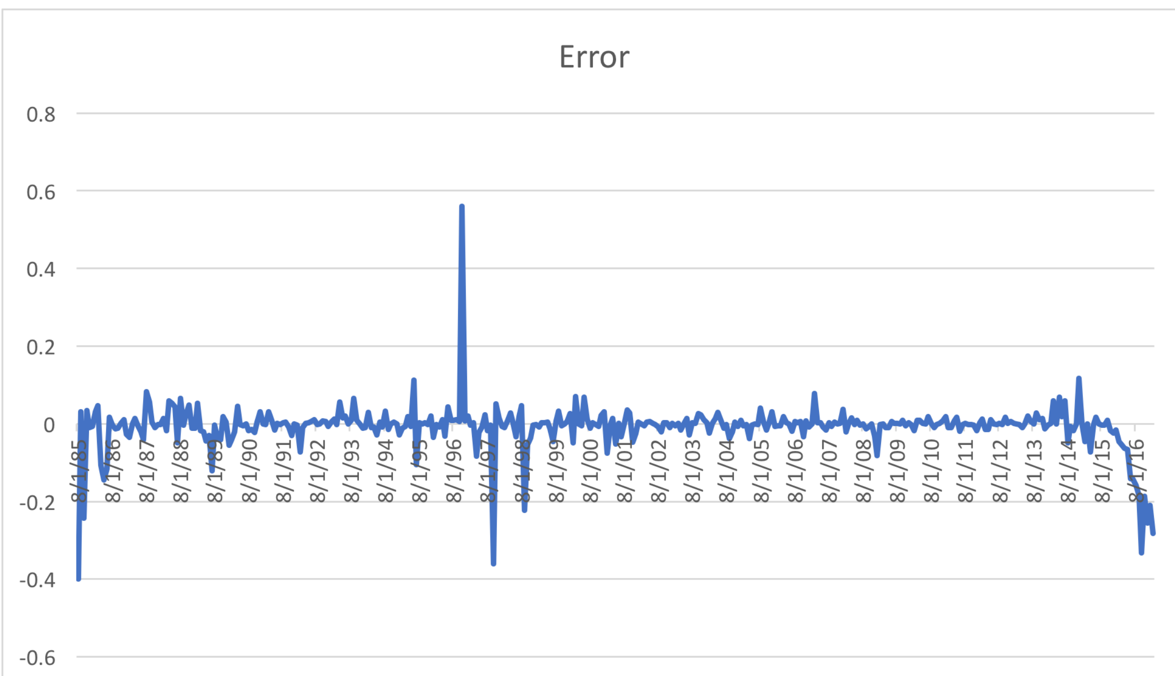

Fig.2

4. An Implication from the Empirical Result

4-1. An Indemnification of Expansion Monetary Policy

Fig.3 shows errors between $\frac{d^3}{dt^3}(f(t))$ and the real exchange rate (RER) in a stage of "input". T and the big errors (above 0.6) are observed in 1985, 1987, 1988, 1996,1997 and after 2013. The NDE includes the term of $\frac{d^3}{dt^3}(f(t))$. In physics, this term is called "jerk". In economics, this term captures a control system, such as a currency policy by the US or Japan government. More concretely, the control power is set by intervention, monetary policy, and political accord[8].

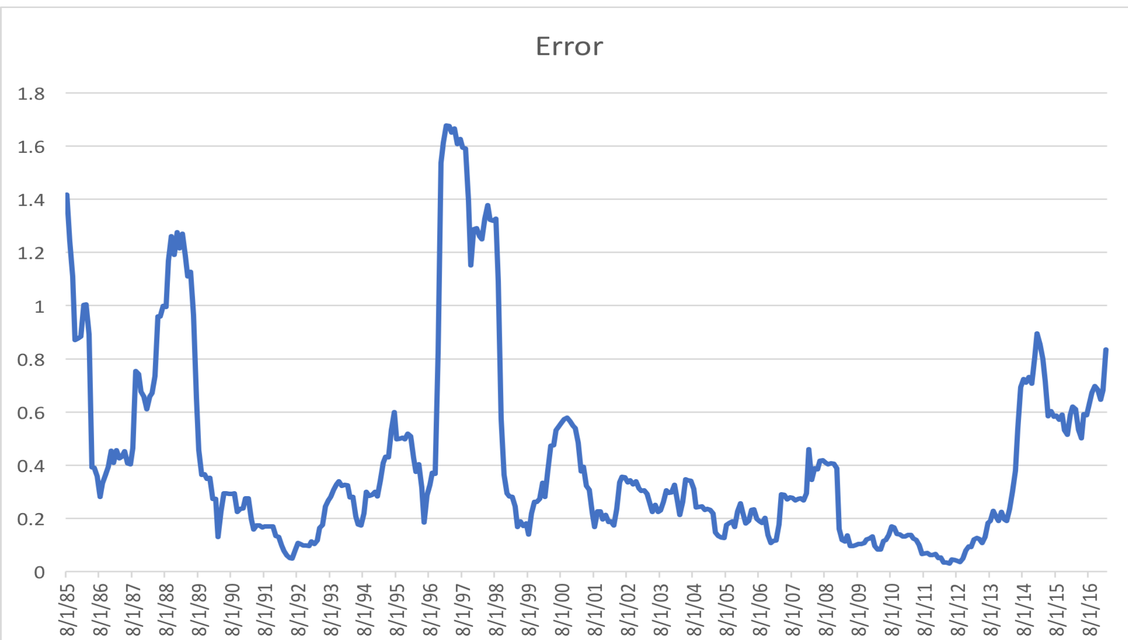

Fig.3

[8] James A. Baker III (2015), the former secretary of the US treasury, says that the middle term and long term exchange rate has been decided by political accords with fundamentals. Broz (2008) also says "Of course, governments cannot directly set the real exchange rate, but they can affect trends in the real exchange rate over a period long enough to be of political and economic significance – typically estimated at three to five years."

In particular, consider the error after 2013. This error is linked to the ultra-losing monetary policy by BoJ, which induced a depreciation of the Japanese yen. The relationship between loosening or tightening monetary policy and depreciating or appreciating currency has been discussed elsewhere; the most familiar paper is Dornbush (1976)[9]. Eichenbaum and Evans (1995) indicates a strong empirical relationship between the federal funds (FF) rate and the US dollar[10]. Brainerd (2016)[11] discusses the relationship between FF rate and the dollar index. Uunconventional monetary policy, like purchasing assets, has also affected the value of the dollar. For instance, Glick and Leduc (2015) find that the monetary policy has roughly three times the bang per policy surprise on the value of the dollar. Caballero, Farhi and Gourienchas (2015) explain why exchange rate movements are more influenced in monetary policy than the Subprime and European Sovereign Debt crises, caused in near-zero interest rate. With regard to speeches by governors and exchange rates, Taylor (2015) discusses ECB President Draghi's speech and the USD/EUR[12].

I discuss the relationship between the BoJ monetary policy and USD/JPY. The Japanese yen, after trading in the 75 yen to 80-yen range to the US dollar in the autumn of 2012, has weakened to the middle 90 yen level amid market expectations for "aggressive monetary easing" under Prime Minister Shinzo Abe's administration. Bernanke (2017) says, "Abe's election the

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[9] Dornbush concludes that a monetary expansion brings temporarily to a large nominal depreciation.
[10] The FOMC on February, 1994 announced a tightening monetary policy from increasing in FF rate from 3% on February 4, 1994 to 4.25% on May 17, 1994 and the dollar depreciated by 10.2% against the yen during February 4 to June 30.
[11] "In particular, estimates from the FRB/US model suggest that the nearly 20 % appreciation of the dollar from June 2014 to January 2016 could be having an effect on U.S. economic activity roughly equivalent to a 200 basis point increase in the federal funds rate," Brainerd (2016) said. "Interestingly, it appears that this effect showed through in decreased business investment activity and stagnant manufacturing output, while the anticipated effect on net exports may have been somewhat dampened by depressed demand for imports of capital goods, among other factors."
[12] At the Jackson Hole conference in August 2014, Mario Draghi spoken about his concerns about the strong Euro and hinted at quantitative easing, which then followed. After this speech, USD/EUR shifted to a weaker Euro and a stronger Dollar.

Bank has effected a substantial easing in financial conditions, as reflected in the stock market, long-term interest rates, and the exchange rate[13]." Bénassy-Quéré (2014) also state that the experience of Japan, whose currency has greatly depreciated since late 2012 following the announcement of a massive expansionary monetary policy, illustrates the link between monetary policy and exchange rates[14].

The error term $\frac{d^3}{dt^3}(f(t)$ is expanded by the Quantitative and Qualitative Monetary Easing 1 (QQE1)[15] on April 4, 2013, and it brings on adjustment against the appreciation of the yen in terms of the EER. Even if the purpose of the policy is to boost competitiveness of Japan's exports, BoJ monetary policy could be evaluated reasonably; the purpose is to escape from deflation of disinflation by depreciating the yen. However, the deflation or disinflation with a 2% inflation target could not be overcome by the QQE 1 only, and BoJ looks upon a factor of deflation or disinflation as a sticky weak demand-side issue[16] (see Fig. 4). Afterwards, BoJ adapted QQE2[17] on October 31, 2014.

By the QQE2, however, not only 2% inflation was not produced, but also only depreciation of yen was induced. As an evidence, the error of the $\frac{d^3}{dt^3}(f(t))$ in a stage of "input[18]" was expanding after February 2014. In my view, the monetary policy doesn't have an moderate tool as a domestic macro-policy for 2% inflation target.

[13] The trade-weighted exchange rate has fallen from 107 to 86 (2010=100).
[14] Neely (2010) says the policies of the Fed also appear to have resulted in a significant but moderate depreciation in the dollar.
[15] https://www.boj.or.jp/en/announcements/release_2013/k130404a.pdf
[16] The weak demand also can be guided. The Japanese Philips curve was steep (downward-sloop) curve in 1970 to 1989: however, the curve in1990s was gentle, 2000s and 2010s was almost flat. The meaning of the flat curve is no inflation with high unemployment, and the implication is the weak demand.
[17] BoJ explains about more committing expansion monetary policy, due to the sticky low inflation.
[18] In the non-linear differential equation, the noise is taken into not only "input" but also "output" account.

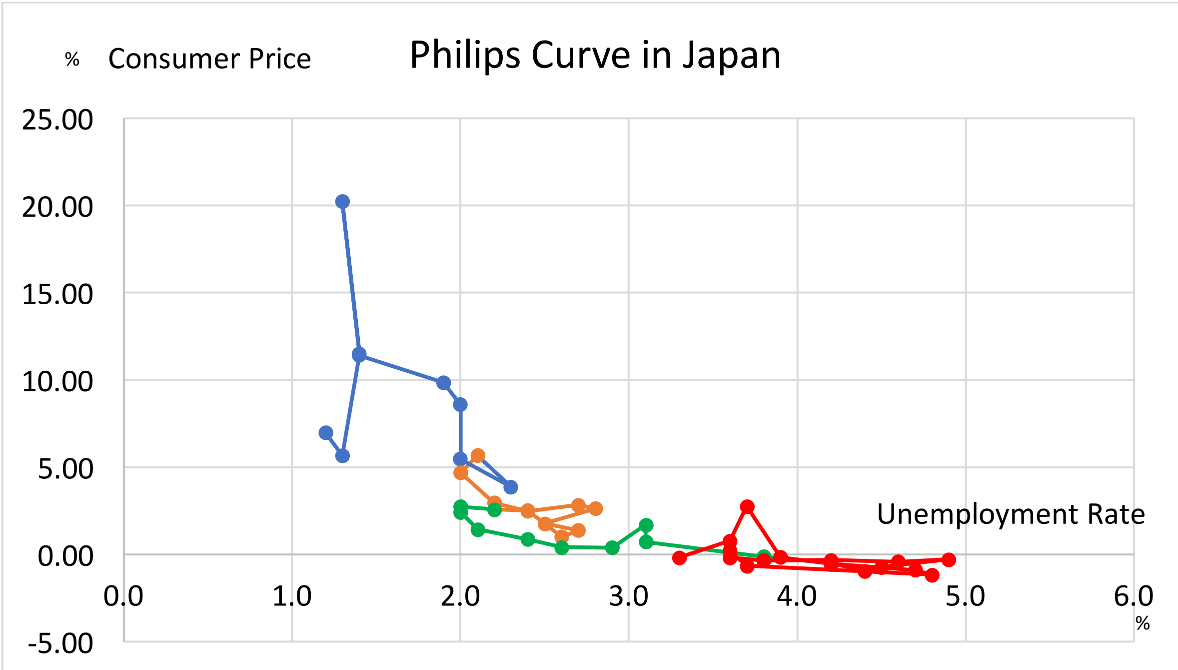

Fig. 4

Note: Blue is 1970s, Orange is 1980s, Green is 1990s, Red is 2000s and 2010s.

Additionally, it casts doubt on currency manipulation including a "beggar-thy-neighbor" policy[19]. Frankel (2015)[20] discusses that trading partner's central bank is free to ease its monetary policy in buying domestic assets if it doesn't like the implications of a dollar depreciation such as resulted in 2010-2011 from Quantitative Easing 2 (QE2). The nearly 40% yen appreciation against the dollar between 2007 and 2012 caused by Fed monetary policy: QE1, QE2, and QE3, however. This does not correspond with currency manipulation. The result is that the error is within range (blow 0.6 (See Fig.3), and the price action is along the Finished Goods Price Index (FGPI) (see Fig. 5 and Appendix C). Even so, the BoJ should contemplate the 2013

[19] Shelton (2017) writes, "There's no denying that one of the primary arrow of Japan's economic strategy under Prime Minister Shinzo Abe, starting in late 2012, was to use radical quantitative easing to boost the "competitiveness" of Japan's exports.

[20] Even foreign interest rates have impact to central bank decisions, for instance. The Norges Bank reported that, in 2010, lowered policy interest rate is lower a foreign interest rate.

G7 communique[21]; monetary policy should not target a currency's depreciation but domestic objectives, like inflation and the output gap.

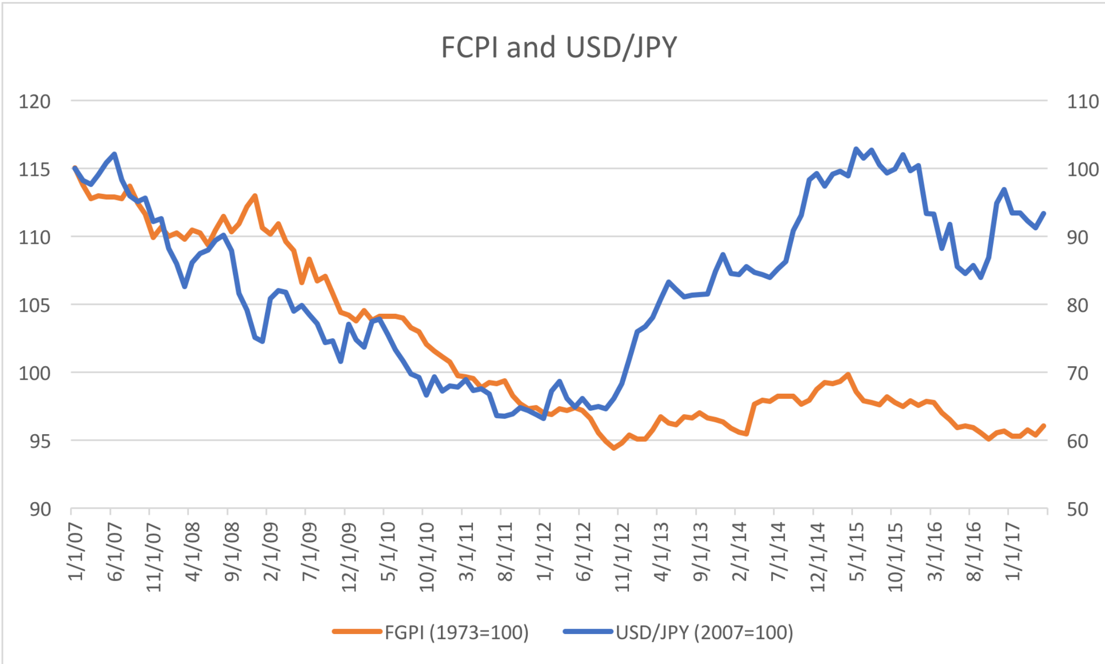

Fig.5

Note: The graph shows the cumulated depreciation (+) or appreciation (-) of the yen against the dollar since January 2007 (=100). About 40%-yen appreciation against the dollar between 2007 and 2012 was entirely reversed following the implementation of Abenomics (April 2013).

4.2 Allowing Difficulty to Movement Along the Equilibrium Exchange Rate

The second discussion is why the real exchange rate (RER) cannot be hovering on an around equilibrium exchange rate (EER) stably. Mr. Beryl Sprinkel, the former undersecretary of

[21] We, the G7 Ministers and Governors, reaffirm our longstanding commitment to market determined exchange rates and to consult closely in regard to actions in foreign exchange markets. We reaffirm that our fiscal and monetary policies have been and will remain oriented towards meeting our respective domestic objectives using domestic instruments, and that we will not target exchange rates. We are agreed that excessive volatility and disorderly movements in exchange rates can have adverse implications for economic and financial stability. We will continue to consult closely on exchange markets and cooperate as appropriate. http://www.mof.go.jp/english/international_policy/convention/g7/g7_130212.pdf

the US Treasury, had the same question. He indicates that the EER was whatever an actual market rate is at this moment. Krugman (1990) states that what advocates of some deliberate policy toward the exchange rate believe is not that there is literal disequilibrium in the market, but something more complex. Frankel and Rose (1995) and Rogoff (2002) state that economic fundamentals are weakly related to exchange rate fluctuations in the short-run, and Rogoff (1996) says, "it is the PPP puzzle[22]."

Why couldn't the RER persist in being equilibrium point such as the EER? There are errors between the real-time series and the NDE model time series. The error consists of a white noise. The errors between RER and NDE model can be depicted by a stochastic differential equation (SDE).

In Appendix. B, the error of the term $\frac{d^3}{dt^3}\big(f(t)\big)$ with "output" is almost zero compared with $\frac{d^3}{dt^3}\big(f(t)\big) + 2\frac{d^2}{dt^2}\big(f(t)\big)$. Accordingly, the factor of the white noise is $\frac{d^3}{dt^3}\big(f(t)\big) + 2\frac{d^2}{dt^2}\big(f(t)\big)$. The SDE is:

$$\frac{d}{dt}\big(f(t)\big) = -2f(t) + \sigma\varepsilon(t) \qquad \text{4-1}$$

Equation 4 -1 is 2nd-order function, where $\varepsilon(t)$ is the white noise. It seems that the white noise is produced by psychology, e.g., technical chart analysis, a statement of central banks or executives, news headlines, a geopolitical risk, or an option price. Fig. 6 shows a potential chart

[22] With regard to how to measure the PPP, there are controversial discussions. "More production costs in Japan are deduced, the lower the price per unit of export leads to yen's EER against the dollar strength," Kazuma (2013) said. "If Japan escapes deflation and Japanese firms come to secure proper margins on par with their U.S. counterparts, an exchange rate of 100 yen or weaker to the dollar may well become justifiable."

for Equation 4-1. The RER is converged finally the point A in Fig. 6, which is an equilibrium exchange rate (EER), is converged finally, but the RER is influenced by the white noise. Thereby, the RER can't hover the EER forever. Before the EER converges the point A, the market considers the next A points which is changed by updated fundamentals. For these reasons the RER cannot stay the EER.

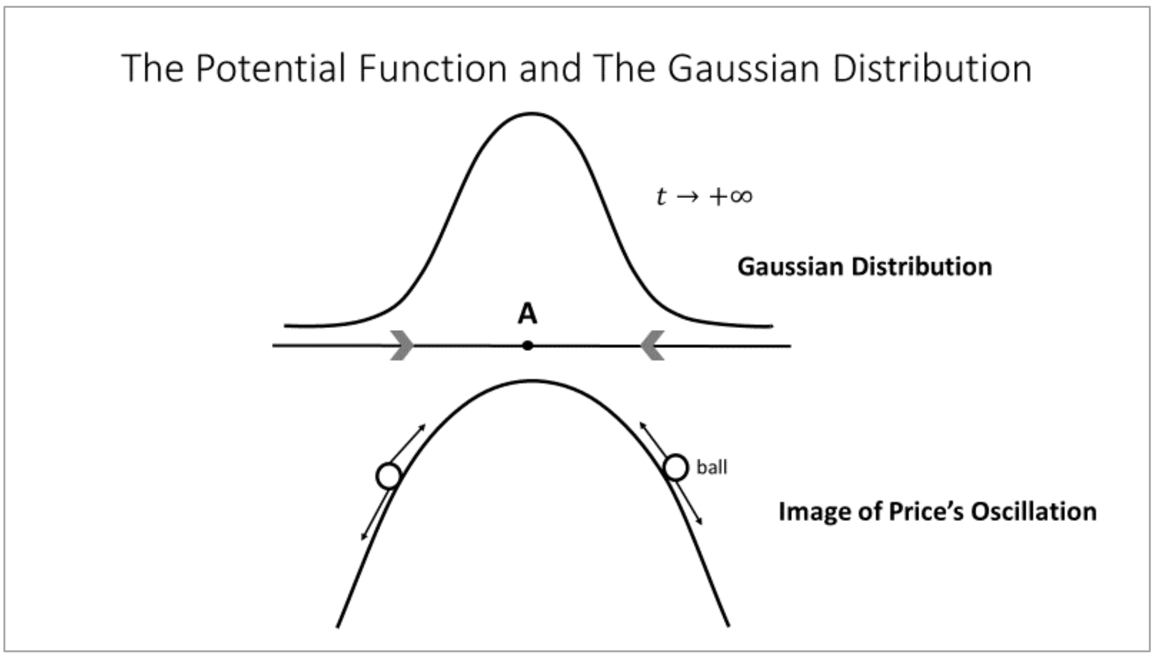

Fig.6

Froot and Obstfeid (1991) describe it through their model: the regime change from a fixed exchange rate system to a floating system caused by the white noise[23], and I apply this discussion to the discuss about whether or not the RER can be shifted a new price range caused by the white noise.

The USD/JPY exchange rate is also influenced by the white noise in their model, and the shifted price range wouldn't go back the EER. However, the exchange rate does eventually

[23] I suggest Engel and Hamilton (1990) conclusion: the factor of the RW doesn't lead to exchange-rate regime.

return to the EER from a shifted price range, because the NDE is described by a 2nd- order potential function with Gaussian distribution. In other words, a ball in Fig. 6 could not run out without changing the form of the potential function.

5. Conclusion

This paper shows that our non-linear differential equation (NDE) model is useful to describe the USD/JPY exchange rate time series. Additionally, I discuss 1) the linkage between monetary policy and the exchange rate, and 2) allowing difficulty to movement along he EER.

From the NDE model, the ultra-losing monetary policy by BoJ leads to a distorted currency value - excessive depreciation of the USD/JPY. Although monetary policy doesn't need to consider currency policy, BoJ should consider the impact on the exchange rate account to avoid triggering a "trade war," with the beggar-thy-neighbor.

Due to the white noise produced by the errors, the real exchange rate (RER) is allowing difficulty to movement along the equilibrium exchange rate (EER). By the influence of the noise, a price range is shifted to another price range. By the noise, however, the RER doesn't deviate a theoretical value such as the PPP because the exchange rate can be dominated by a currency policy.

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Appendix. A

The L^p-norm of a function f is

$$\|f\|_p = \left(\int_{-1}^{1} |f(x)|^p dx\right)^{1/p}$$

This is defined for $1 \leq p < \infty$ and for any measurable f. The function space $L^p[-1, 1]$ is the class of measurable functions for which the above norm is finite. The norm $\|f\|_\infty$ of a measurable function f is the *essential supremum*. Roughly speaking, this is the largest value of $|f(x)|$ if you ignore sets of measure zero. It turns out to be the limit of the norms $\|f\|_p$ as p tends to infinity. The space $L^\infty[-1, 1]$ consists of those measurable functions f for which $\|f\|_\infty$ is finite. While the L^∞ norm is concerned solely with the "height" of a function, the L^p norms are instead concerned with a combination of the "height" and "width" of a function.

Particularly important among these norms is the L^2-norm, since $L^2[-1, 1]$ is a Hilbert space. This space is exceptionally rich in symmetries: It admits a wide variety of *unitary transformations*, that is, invertible linear maps T defined on $L^2[-1, 1]$ such that $\|Tf\|_2 = \|f\|_2$ for every function $f \in L^2[-1, 1]$ (See Gowers 2008).

Appendix. B

I show the general solution of the differential equation.

$$f(t) = (f'''(t))^2 + 2f'''(t) - f''(t) \qquad -(1)$$

Putting

$$f(t) = f'''(t)(f'''(t) + 2f''(t)) \qquad -(2)$$

$f'''(t) < f'''(t) + 2f''(t)$, because of comparison of error $f'''(t)$ with a stage of “output” with a real-time series data.

Therefore

$$f'''(t) + 2f''(t) = \varepsilon_t \qquad -(3)$$

Integrating equation (3) twice with respect to time yields

$$f'(t) = -2f(t) + \varepsilon_t \qquad -(4)$$

Thus $f(t) = Y \cdot e(-2t)$, by the general solution of $f'(t) + 2f(t) = 0$.

Equation (4) is

$$Y' \cdot e(-2t) = \varepsilon_t$$

$$Y = \sum_{k=1}^{t} e(2k) \cdot \varepsilon_k \qquad -(5)$$

Therefore

$$f(t) = e(-2t) \cdot \sum_{k=1}^{t} e(2k) \cdot \varepsilon_k \qquad -(6)$$

Appendix. C

The Finished Goods Price Index (FGPI) is based on the Purchasing Power Parity (PPP), which compares different countries' currencies through a market "basket of goods" approach. More precisely,

$$EX = \frac{P_1}{P_2}$$

where EX represents the USD/JPY exchange rate, P_1 represents a monthly change of PPI (Producer Price Index) Finished Goods in the US, and P_2 represents a monthly change of PPI in Japan. The FGPI value is 1973=100. Data are obtained from Bloomberg.

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