Title:

The burden of public debt in neoclassical growth models: Do we have to worry about it?

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Abstract:

This paper investigates the crowding-out effect of public debt and the related loss in the long-run output in the framework of neoclassical growth models. To accomplish this task, we incorporate the government sector into three basic neoclassical models, which differ only in their assumptions about the consumption behavior of households. First, we consider the crowding-out effect of public debt in the Ramsey-Cass-Koopmans (RCK) model with dynamic optimization and the altruistic intergenerational links of households. Then, we drop the assumption of intergenerational links in the Blanchard (1985) model and later the assumption of dynamic optimization as well in the Solow model. Our results show that, contrary to the RCK model, public debt reduces long-run output in the Blanchard model and the Solow model, although to a different extent: the crowding-out effect is marginal in the former, whereas it can be very large in the latter depending on the households' saving rate and the population growth rate. However, we demonstrate that under the conditions of developed countries, even the upper limit of the output loss triggered by public debt is moderate at best. This conclusion holds even if the output loss resulting from distortionary taxes is taken into account. Our main policy contribution is that according to the neoclassical growth models, the long-run burden of public debt around the current 90 percent average debt-to-GDP ratio seems to be of minor importance in the Eurozone.

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Running title:

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1. Introduction

This paper investigates the burden of public debt in neoclassical growth models. Our goal is to quantify the crowding out of physical capital by public debt and the related loss in long-run output under different assumptions regarding the consumption behavior of households. The relevance of the issue can be traced back to the historical indebtedness of developed countries and the debate on the euro area's crisis management.

In the wake of the global financial crisis, the majority of the developed world slipped into deep recession in 2009. The time elapsed since then has been naturally devoted to the management of the crisis. Initially, the crisis management was based on Keynesian economics in each country and operated with expansionary fiscal policy to stimulate economic growth and employment. This countercyclical fiscal policy, complemented by the necessary recapitalization of the banking sector in some countries, placed government budgets under considerable pressure. As a result, public debt has begun to rise steeply everywhere (Figure 1) evoking the sovereign debt crisis of the euro area in the spring of 2010 (Lane, 2012).

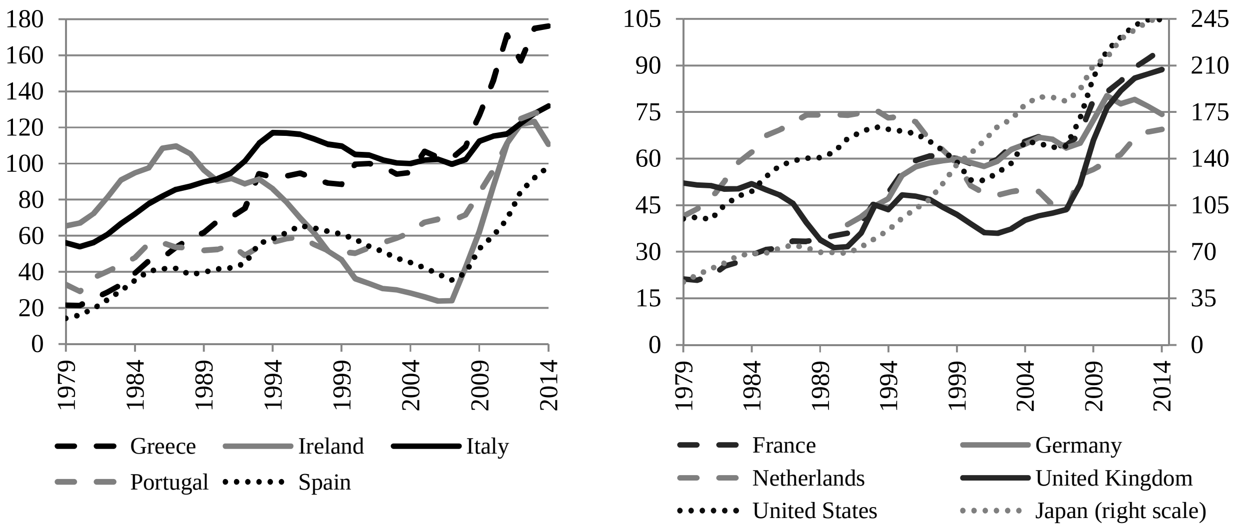
The debt crisis triggered a switch in the stance of European crisis management: the expansionary fiscal policy focusing on the stimulation of the economy has been replaced by overall fiscal austerity focusing on the reduction of the debt-to-GDP ratio. According to the general belief at that time, the greatest danger for the European recovery was high public debt. Thus, the right way for economic policy to foster economic growth was to stop the growing indebtedness of the individual countries. This view was supported both by academics (Alesina and Ardagna, 2010; Reinhart and Rogoff, 2010) and high-ranked officers of the euro area.[1] On the contrary, the USA has maintained the expansionary fiscal and monetary policy mix to help the recovery of its economy.

Many years have passed since the outbreak of the crisis, and the euro area is still struggling with high unemployment and low growth. Meanwhile, the USA is forging ahead with continuously rising GDP and employment. Moreover, the theoretical pillars of the austerity policy – that is, the expansionary austerity and the 90 percent debt-to-GDP threshold – have been seriously questioned in recent years (Blanchard and Leigh, 2013; Eberhardt and Presbitero, 2015). Thus, it is no surprise that the quest for fiscal easing – mainly in the form of additional public investments – in countries with enough fiscal space (e.g., Germany) has re-emerged to jumpstart the euro area economy (IMF, 2014, 2015; Stiglitz et al. 2014).

[1] Interview with Jean-Claude Trichet, President of the ECB. Conducted by Elena Polidori on 16 June 2010.

http://www.ecb.europa.eu/press/key/date/2010/html/sp100624.en.html

Figure 1. The ratio of public debt to the GDP in some developed countries (%)



Source: AMECO Database

Notes: In Japan the debt-to-GDP ratio was 246.3 percent in 2014. For the sake of transparency, this data point is omitted.

With this economic background, the revival of the interest in the *'public debt – economic growth nexus'* has two reasons. First, there is much concern that the historically high public debt-to-GDP ratios, which are not expected to decrease significantly in the foreseeable future, will have adverse effects on growth prospects (Reinhart and Rogoff, 2013). Second, a common argument against fiscal easing in the core euro area countries is the possible negative effect of public debt on economic growth.

The theory does not provide a clear-cut relationship between public debt and economic growth. There are many channels through which public debt might affect economic output either positively or negatively, depending mainly on the stance of the business cycle. The most frequently cited negative consequence of public debt is the crowding out of private investments (Elmendorf and Mankiw, 1999). A further adverse effect is the macroeconomic vulnerability of highly indebted countries. Two major positive effects of public debt are the Keynesian effect and the hysteresis effect, which refer to the ability of expansionary fiscal policy to mitigate both the actual rate and the natural rate of unemployment during recessions (DeLong and Summers, 2012). An additional positive impact can arise if the public debt temporarily finances productive expenditures (e.g., infrastructure, education) that increase the long-run output of the economy.

Not just the various channels of the debt-growth relationship, but the conditionality of these channels on the business cycle, the institutions and the debt history also complicate our understanding of the issue. In times of economic boom, the negative effects, while in times of economic slump, the positive effects of public debt dominate (Krugman, 2012).[2] Furthermore, it is also well known that debt tolerance is much larger in developed countries with better institutions and debt history than in developing ones (Reinhart et al., 2003).

The main message of economic theory is that the impact of public debt on economic growth is country- and time-specific. This finding is confirmed by the latest results of the empirical literature as well (Égert, 2015; Eberhardt and Presbitero, 2015; Dreger and Reimers, 2013; Kourtellos et al., 2013).[3]

Given the importance and the relevance of the issue, it is urgent that economic theory improves our understanding of the complex relationship between public debt and economic growth. Our paper aims to contribute to this mission by thoroughly investigating the magnitude of the crowding-out effect of public debt under different consumption behaviors. The framework of the analyses is provided by three basic types of neoclassical growth models: the Ramsey-Cass-Koopmans (henceforth RCK) model (Ramsey, 1928; Cass, 1965; Koopmans, 1965), the Blanchard model (Blanchard, 1985) and the Solow model (Solow, 1956). We start with the RCK model and assume that households pursue dynamic optimization

[2] For example, the crowding-out effect becomes completely irrelevant in a balance sheet recession with a liquidity trap and with an intensively deleveraging private sector (Eggertsson and Krugman, 2012).

[3] The empirical literature has focused almost exclusively on the 90 percent 'magic' threshold of Reinhart and Rogoff (2010) in the last few years. According to these authors, economic growth decreases significantly beyond the 90 percent debt-to-GDP ratio. This finding was confirmed by some early papers (e.g., Checherita-Westphal and Rother, 2012). However, Eberhardt and Presbitero (2015) prove that statistically there is no unique threshold in the debt-growth relationship and those papers which seemed to verify it are based on inappropriate econometric methodology.

and are connected by altruistic intergenerational links. The RCK model and its implications concerning the effect of public debt are well-understood in the literature, so we discuss them only to provide a theoretical baseline for the subsequent analyses. After the RCK model, we drop the assumption of intergenerational links in the framework of the Blanchard model and later the assumption of dynamic optimization as well in the framework of the Solow model. In each case, we consider a closed economy and focus on the long run, so we neglect the transitional dynamics throughout the paper.

We are not the first to investigate the crowding-out effect of public debt in neoclassical growth models. Indeed, there is a long history of this vein of the literature.[4] We quote only the major milestones here. In his seminal paper, Diamond (1965) deals with the effect of public debt in a life-cycle OLG model. Based on the Diamond model, Auerbach and Kotlikoff (1987) simulate the short- and long-run effects of different fiscal policies and reveal significant crowding out of private investments by public debt. Inspired by the results of life-cycle models, Barro (1974) demonstrates that if intergenerational links prevail, government bonds do not represent net wealth for the households and therefore Ricardian equivalence holds. Blanchard (1985) constructs an OLG model which neglects the life-cycle aspect of life in a continuous time setting in order to provide a tractable framework for analyzing the effect of public debt on long-run output when the time horizon of households is finite. Weil (1989) and Buiter (1988) reveal that the failure of Ricardian equivalence in the basic Blanchard model is caused by the disconnectedness of new and old dynasties and not by the finite time-horizon of households. Ball and Mankiw (1995) introduce the parable of the debt fairy, a back-of-the-envelope type calculation, into the literature in order to calculate the proximate burden of public debt in the Solow model.

The main contribution of the paper to the *growth-debt debate* is to provide an overview on the magnitude of the crowding-out effect as a function of households' behavior in a neoclassical growth

[4] The other traditional vein of the debt-growth literature investigates the issue in the frame of endogenous growth theory. The related seminal papers are Barro (1990) and Saint-Paul (1992). For some recent results see e.g. Yakita (2008), Greiner (2008, 2012), Greiner and Fincke (2009), Teles and Mussolini (2014) and Broner et al. (2014).

framework. Although, our knowledge about consumption and saving has improved a lot in the last two decades, there is still considerable confusion about the extent to which intergenerational links (generational disconnectedness) and dynamic optimization (myopia) might characterize the behavior of households.[5] Thus, to provide an approximate range on the possible burden of public debt is of first-order importance from the point of view of current economic policy. Our results show that in a neoclassical world the long-run output loss related to public debt is moderate at best. Beyond this major policy conclusion, the paper provides two additional contributions to the theory as well. First, it is often argued that the burden of public debt through distortionary taxation can be considerable (e.g., Mankiw 2000). We prove that in fact this is not true, at least in the RCK model. Second, we present a new formula for the crowding-out effect in the Blanchard model according to which it is straightforward to perform the calculation.

The remainder of the paper is organized as follows. Section 2 discusses briefly the role of public debt in the RCK model. Section 3 considers the crowding-out effect and the resulting output loss in the Blanchard model, while section 4 discusses them in the Solow model. Section 5 concludes the paper.

2. Government debt in the Ramsey-Cass-Koopmans model

In this section, we investigate the impact of government debt on steady-state output in the Ramsey-Cass-Koopmans model (Ramsey, 1928; Cass, 1965; Koopmans, 1965).[6] We consider a closed economy that consists of three sectors: households, firms, and government.

The representative household supplies labor inelastically and decides only on consumption. As intergenerational links are operative, households maximize utility on the infinite time horizon:

$$\max_{c(t)} U = \int_0^\infty u\left[c(t)\right] e^{nt} e^{-\rho t} dt \quad , \qquad (1)$$

[5] For a discussion, see Romer (2012).

[6] The RCK model is well-known in the literature (see, e.g., Romer, 2012, ch.2), so we sketch it only to the necessary extent. The full model is provided upon request. In what follows, we use ‘steady state’ and ‘long run’ as synonyms.

where ρ is the subjective discount rate, $c = C / L$ is the per capita consumption, C is the aggregate consumption, $L(t) = e^{nt}$ is the population growing at rate n and $\rho > n$. The initial value of the population is normalized to one. For simplicity, we assume the logarithmic utility function: $u(c) = \ln(c)$.[7]

The households' flow budget constraint is

$$\dot{a}(t) = (1-\tau_W)w(t) + (1-\tau_A)r(t)a(t) - na(t) - c(t) \quad . \tag{2}$$

where a and w are the per capita assets and wage, respectively, r is the interest rate, and τ_W and τ_A are the tax rates levied on wage income and capital income. The dot above the variables denotes derivation with respect to time. Because the economy is closed, the total assets ($A = aL$) equal the sum of physical capital (K) and government debt (B): $A = K + B$.

The intensive form of the $Y = F(K, EL)$ neoclassical production function is $\hat{y} = f(\hat{k})$, where $\hat{y} = Y/(EL)$, $\hat{k} = K/(EL)$, Y is output, and $E = e^{gt}$ is the level of technology growing at a constant rate of g.

Firms are supposed to operate in competitive markets; thus, production factors are rewarded by their marginal products:

$$r + \delta = \partial f(\hat{k}) / \partial \hat{k} = f'(\hat{k}) \text{ and } \hat{w} = f(\hat{k}) - \hat{k} f'(\hat{k}) \quad , \tag{3}$$

where δ is the depreciation rate of physical capital and $\hat{w}$ is the wage per effective labor.[8]

The government collects revenues by imposing taxes on labor and capital income, while its outlays consist of government expenditures and interest payments on debt. For simplicity, the consumption tax is neglected. Thus, the government obeys the following flow budget constraint:

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[7] The results are robust to this assumption and hold with general CIES (constant intertemporal elasticity of substitution) utility function as well.

[8] In the remainder of the paper, we neglect the t time index when no confusion emerges.

$$\dot{B}(t) = r(t)B(t) + G(t) - \tau_W W(t) - \tau_A r(t)(K(t) + B(t)) = r(t)B(t) - \Gamma_{RCK}(t) \quad , \tag{4}$$

where G is government expenditures, W is aggregate wages, and $\Gamma_{RCK} = \tau_W W + \tau_A rA - G$ is the primary balance of the budget.

For simplicity, government expenditures are assumed to affect neither the utility of households nor the production of firms. Government expenditures as a share of GDP ($\phi = G(t)/Y(t)$) are considered to be constant, therefore tax rates are also constant for any given debt-to-GDP ratio. In other words, tax rates are set to run the necessary primary budget surplus in order to achieve the target of the government for the long-run debt-to-GDP ratio. Of course, the latter implies that larger government debt results in higher tax rates. This increase of the tax rates raises the well-known issue of distortionary taxes stemming from higher public debt.

The crowding out of capital brought about by distortionary taxes and the related output loss must be treated separately from the classical crowding-out effect of public debt. Although the rising tax rates crowd out capital indirectly, the mechanism is completely different from the classical crowding-out effect of public debt which effect is related to the consumption-saving behavior of households and the wealth effect of government bonds. Consequently, we do not deal with distortionary taxes triggered by public debt in the main text of the paper. Nevertheless, an appendix is devoted to the issue, especially as to some authors "…substantial steady-state crowding out can occur simply because of distortionary taxation." (Mankiw, 2000 pp. 123.). However, in the appendix we demonstrate that the output loss triggered by public debt through the channel of tax distortions is quantitatively rather small therefore it cannot causes serious concerns.

The steady state

The representative household maximizes utility (eq.1) subject to its budget constraint (eq.2). According to the first-order conditions of the dynamic optimization, the time path of consumption per effective labor ($\hat{c} = C/(EL)$) is

$$\dot{\hat{c}}/\hat{c} = (1-\tau_A)r - \rho - g \quad . \tag{5}$$

The dynamics of physical capital per effective labor can be derived according to the households' budget constraint, taking into account that $\dot{a} = \dot{k} + \dot{b}$, where $b = B/L$, and using equations (3) and (4):

$$\dot{\hat{k}} = (1-\phi) f(\hat{k}) - \hat{c} - (n+g+\delta)\hat{k} \quad . \tag{6}$$

The dynamics of the system are determined by equations (4), (5) and (6). In steady state, $\hat{c}$, $\hat{k}$ and $\hat{b} = B/(EL)$ are constant, which yield the determining equations of the long-run equilibrium:[9]

$$\dot{\hat{c}}^* = 0 \rightarrow (1-\tau_A)\left(f'(\hat{k}^*) - \delta \right) = \rho + g \quad , \tag{7}$$

$$\dot{\hat{k}}^* = 0 \rightarrow \hat{c}^* = (1-\phi) f(\hat{k}^*) - (n+g+\delta)\hat{k}^* \quad , \tag{8}$$

$$\dot{\hat{b}}^* = 0 \rightarrow \frac{\hat{b}^*}{\hat{y}^*} = \mu^* = \frac{\Gamma^*_{RCK}/Y^*}{r^* - g - n} \quad , \tag{9a}$$

where the asterisk refers to the steady state and μ is the debt-to-GDP ratio.

Another useful formula to express the steady-state debt-to-GDP ratio, which we will intensively use up in section 4, is

[9] According to equation (4), the dynamics of $\hat{b}$ is clear-cut: $\dot{\hat{b}} = (r-g-n)\hat{b} - \Gamma_{RCK}/EL$.

$$\mu^* = -\frac{\eta^*}{g_Y^*} \quad , \tag{9b}$$

where η is the total balance of budget divided by the GDP and $g_Y^* = g + n$ is the steady-state growth rate of output.[10]

The intertemporal budget constraint of households

The crowding-out effect of public debt depends on how consumption reacts to the government budget deficit. To answer this question, we must focus on the intertemporal budget constraint of the households and the government.

Based on the solution of the flow budget constraint and the transversality condition of the dynamic optimization, the intertemporal budget constraint of the representative household is

$$\int_0^\infty c(t) e^{-[(1-\tau_A)\bar{r}(t)-n]t} dt = a(0) + \int_0^\infty (1-\tau_W) w(t) e^{-[(1-\tau_A)\bar{r}(t)-n]t} dt = a(0) + \bar{W} - \bar{T}_W \quad , \tag{10}$$

where $\bar{r}(t) = \int_0^t r(s)ds \big/ t$, $\bar{W}$ is the present value of the wage income and $\bar{T}_W$ is the present value of the taxes levied on the wages. Regarding the fact that the value of any asset equals the present value of its future net incomes, the initial stock of assets can be written as

$$a(0) = k(0) + b(0) = \int_0^\infty \left(R_K(t) + R_B(t) - T_K(t) - T_B(t) \right) e^{-(1-\tau_A)\bar{r}(t)t} dt = (\bar{R}_K - \bar{T}_K) + (\bar{R}_B - \bar{T}_B), \tag{11}$$

where R_i is the income earned on asset i $(i = K,\ B)$, T_i is the tax imposed on R_i, and $\bar{R}_i$ and $\bar{T}_i$ are the present values of the respective future asset incomes and taxes.

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[10] Focusing on the real economy, we neglect inflation throughout the study. As a result, the interest rate in the debt equations represents the real interest rate. However, this does not influence our results. The only thing we should change if we took inflation into account is that η would stand for the operational (inflation-adjusted) rather than the total balance of budget.

Substituting equation (11) into equation (10), the intertemporal budget constraint can be rewritten as follows:

$$\int_0^\infty c(t)e^{-[(1-\tau_A)\bar{r}(t)-n]t}dt = k(0)+b(0)+\bar{W}-\bar{T}_W = (\bar{W}+\bar{R}_K+\bar{R}_B)-(\bar{T}_W+\bar{T}_K+\bar{T}_B) \quad . \qquad (12)$$

Equation (12) states that the present value of future consumption must equal the present value of all types of income net of taxes.

The government's intertemporal budget constraint

To set up the government's intertemporal budget constraint, one has to follow the same line as in the case of households. Solving the differential equation of the flow budget constraint (eq.4), then taking the limit of the solution and using the no-Ponzi-game (NPG) condition, one arrives at the intertemporal budget constraint of the government:[11]

$$B(0) = \int_0^\infty e^{-(1-\tau_A)\bar{r}(t)t}[\tau_W W(t)+\tau_A r(t)K(t)-G(t)]dt = \bar{T}_W+\bar{T}_K-\bar{G} \quad , \qquad (13)$$

where $\bar{G}$ is the present value of future government spending. Equation (13) claims that the initial public debt must equal the present value of future primary budget surpluses exclusive of the tax revenues from interest payment on government bonds $(\tau_A rB)$. As the initial value of population is normalized to one, the equation also determines the initial per capita debt $(b(0)=B(0))$. It is worth noting that the constraint does not rule out policies that cause the debt to grow permanently. Moreover, if the debt-to-GDP ratio is stable, then the government's intertemporal budget constraint is satisfied in a dynamically efficient economy regardless of the level at which the stability occurs (Barro, 1976; Greiner, 2011).[12]

[11] The NPG condition pins down that debt must grow slower than the net interest rate paid on debt: $\lim_{t\to\infty} B(t)e^{-(1-\tau_A)\bar{r}(t)t} = 0$.

[12] With regard to the sustainability of public debt, the intertemporal budget constraint has quite minor practical relevance. For a short demonstration, consider the following. In the long run, the debt-to-GDP ratio stabilizes at the ratio of the

The effect of public debt on steady-state output

To find out how public debt affects steady-state output, we must combine the government's intertemporal budget constraint with that of the representative household. For doing so, substitute equation (13) for *b(0)* in equation (12):

$$\int_0^\infty c(t)e^{-[(1-\tau_A)\overline{r}(t)-n]t}dt = \overline{W} + \overline{R}_K - \overline{G} \quad , \qquad (14)$$

Equation (14) conveys the key results of the RCK model with regard to the impact of public debt on steady-state output. First, the present value of future incomes stemming from government bonds disappears from the intertemporal budget constraint that is government bonds do not represent net wealth for households (Barro, 1974). Second, government purchases enter in place of taxes.

If taxes were lump-sum, the preceding two points would imply that only the magnitude of government purchases, but not their financing method, affects the consumption of households and thereby the steady-state output.. Consequently, the Ricardian equivalence holds.

If taxes are proportional, as in our case, the above considerations continue to be valid almost completely. As equation (14) shows, government bonds still do not represent net wealth for households. In other words, the additional income from a deficit financed tax reduction does not affect directly the present value of households' lifetime income. However, the financing of government

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primary balance of budget (relative to GDP) to the (*interest rate – growth rate)* difference (eq.9a). Because the upper bound for (Γ^* / Y^*) is one, it is possible from a theoretical point of view that the debt-to-GDP ratio stabilizes at such a high level that is very far from the observed data. For example, setting the difference between r^* and g_Y^* to 10 percentage points, which is extraordinarily high, the upper limit for a stable debt-to-GDP ratio is 1000 percent. Although a debt-to-GDP ratio of this magnitude is certainly not sustainable, the intertemporal budget constraint of the government is not violated, because debt grows more slowly than the net interest rate paid on debt. (Note, that in steady state, the net interest rate exceeds the growth rate of output (eq.7)) Thus, the intertemporal budget constraint is a rather weak assumption for the sustainability of public debt (Ghosh et al., 2013).

expenditures is not neutral anymore. If government expenditures are financed temporarily by budget deficit, tax rate will be higher after the debt-to-GDP ratio stabilizes again at its new long-run value. Furthermore, it is obvious that the higher tax rate results in higher interest rate and thus in lower steady-state capital and income (eq.7). It is important to emphasize that in the latter case the crowding out of physical capital occurs via tax distortions and not via the wealth effect of government bonds. We demonstrate in the appendix that the output loss triggered by distortionary taxes stemming from government debt amounts only to few percentage points, even in the case of a 90 percent debt-to-GDP ratio featuring the Eurozone average recently.

To conclude the section, we assess, that public debt does not crowd out physical capital through the classical channel in the RCK model since the Ricardian equivalence holds. However, it does reduce the steady-state capital and output due to distortionary taxation, but the magnitude of this kind of crowding out is not significant at all. Accordingly, the distortionary taxation does not alter the central message of the RCK model: the burden of public debt is rather small in this model.

3. Public debt in the Blanchard model

The results of the RCK model depend crucially on the assumed behavior of households. However, intergenerational links and dynamic optimization can both be challenged by the empirics. First, "…many people leave no bequest and, therefore, are not economically linked to future generations." (Mankiw, 1995 pp. 279.). Moreover, even if bequests are present, they are unintended in many cases; that is, altruism does not play a role in determining them (Bernheim, 1987). Second, the latest global financial crisis provides overwhelming evidence that expansionary fiscal policy has a significant impact on economic performance in severe recession, which is in sharp contradiction with the Ricardian equivalence (Blanchard and Leigh, 2013). Thus, in this section, we drop the assumption of intergenerational links and investigate the effect of public debt on long-run output in the OLG model of Blanchard (1985) with finite time horizon.

The structure of the economy is the same as in the RCK model. We continue to suppose a closed economy with three sectors: 1. government with constant tax rates and with expenditures influencing neither the production of firms nor the utility of households, 2. firms with neoclassical production function and operating in competitive markets, and 3. households with dynamic optimization. The only difference with regard to the RCK model is the lack of intergenerational links. In OLG models, individuals do not care about the utility of their descendants and make consumption decisions solely with respect to their own life-cycle. This means that individuals optimize their consumption on finite time horizons.[13] This change in household behavior leads to qualitatively important consequences with regard to the impact of public debt on steady-state output. We only introduce the main structure of the model, incorporating the government sector from the very beginning.[14]

Let p be the probability of death per unit of time, which is independent of age. Then, the probability of a person (or household) born at time j being alive at time $t > j$ is $e^{-p(t-j)}$. The expected lifetime is $1/p$. The population grows at a constant n rate, so $L = e^{nt}$. Given this and the p death rate, the size of the cohort born at time t is $(n+p)e^{nt}$.

Regarding the capital market, the Blanchard model assumes that all savings are held in the form of life insurance that pays an annuity (z) over the riskless interest rate (r). If the individual dies, his assets will be left to the life insurance company. The expected profit of a life insurance company at time t with respect to an individual born at time j and with assets $a(j,t)$ is $\pi(a(j,t)) = p \cdot a(j,t) - z \cdot a(j,t)$. Thus the zero-profit requirement of competitive markets ensures that the annuity must equal the probability of death: $z = p$. This implies that the rate of return on households' assets is $(r + p)$.

The individual maximizes the expected utility of lifetime consumption as follows:

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[13] Because of the lack of intergenerational links, one can think of households as consisting only of one member.

[14] The full model is provided upon request. For additional discussion of the Blanchard model with and without government sector, see Blanchard (1985), Acemoglu (2009, ch.9.8) and Barro and Sala-i-Martin (2004, ch.3.6).

$$\max_{c(j,v)} E\left[U(t)\right] = \int_t^\infty ln\left[c(j,v)\right] e^{-(\rho+p)(v-t)} dv \quad , \tag{15}$$

where *E[.]* refers to the expected value. The term $e^{-p(v-t)}$ in equation (15) is the probability of being alive at time *v* provided that the person was alive at time *t.*

The households' budget constraint is similar to equation (2) with two exceptions. First, the budget constraint does not contain the growth rate of population because of the absence of intergenerational links. Second, the returns on assets are the sum of the interest rate and the annuities. Thus, we have

$$\dot{a}(j,v) = (1-\tau_A)(r(v)+p)a(j,v) + (1-\tau_W)w(v) - c(j,v) \quad . \tag{16}$$

Given equations (15) and (16), the Hamiltonian can be set up, and using the first-order conditions for a maximum of expected utility, we obtain the dynamics of consumption for the individual:

$$\frac{\dot{c}(j,v)}{c(j,v)} = (1-\tau_A)(r(v)+p) - (\rho+p) = r_{ne}(v) - (\rho+p) \quad , \tag{17}$$

where $r_{ne}(v) = (1-\tau_A)(r(v)+p)$ is the net effective interest rate at time *v*, which adjusts both for the tax rate and the risk premium resulting from death. Solving equation (16) and taking into account the transversality condition of the dynamic optimization, we obtain the intertemporal budget constraint of households:

$$\int_t^\infty c(j,v) e^{-\overline{r}_{ne}(v)(v-t)} dv = a(j,t) + \int_t^\infty (1-\tau_W) w(v) e^{-\overline{r}_{ne}(v)(v-t)} dv \quad , \tag{18}$$

where $\overline{r}_{ne}(v) = \int_t^v r_{ne}(s)ds \Big/ (v-t)$. Solving equation (17) and substituting for *c(j,v)* in equation (18), we arrive at the present consumption of the individual born at time *j*:

$$c(j,t) = (\rho+p)(a(j,t) + (1-\tau_W)\overline{w}(t)) \quad , \tag{19}$$

where $\overline{w}(t)$ is the present value of the individual's future wage incomes.

Aggregating equation (19) over the population and differentiating it with respect to time yields the dynamics of aggregate consumption per effective labor:[15]

$$\frac{\dot{\hat{c}}}{\hat{c}} = r_{ne} - (g+\rho+p) - (\rho+p)(p+n)\frac{\hat{a}}{\hat{c}} \quad . \tag{20}$$

In steady state, $\dot{\hat{c}} = 0$; thus, equation (20) provides the following relationship between steady-state consumption and assets $(\hat{c}^*,\ \hat{a}^*)$:

$$\hat{c}^* = \frac{x\hat{a}^*}{r_{ne}^* - (g+\rho+p)} = \frac{x\hat{a}^*}{(1-\tau_A)(f'(\hat{k}^*)-\delta+p)-(\rho+p)-g} \quad , \tag{21}$$

where $x = (\rho+p)(p+n)$ and $\hat{a}^* = \hat{k}^* + \hat{b}^*$.

The finite time horizon of households modifies not just the equation of motion for the aggregate consumption, but the equation of motion for the public debt as well compared to the RCK model:

$$\dot{B}(t) = r(t)B(t) + G(t) - \tau_W W(t) - \tau_A(r(t)+p)(K(t)+B(t)) = r(t)B(t) - \Gamma_B(t) \quad , \tag{22}$$

where $\Gamma_B = \tau_W W + \tau_A(r+p)(K+B) - G$. The only difference between equations (4) and (22) is the additional tax revenue term $(\tau_A pA)$ in equation (22) thanks to the higher rate of return on assets in the Blanchard model. Following the same steps as in section 2, the intertemporal budget constraint can be expressed as

$$B(t) = \int_t^\infty e^{-(\overline{r}_{ne}(v)-p)v}[\tau_W W(v) + \tau_A(r(v)+p)K(v) - G(v)]dv \quad .$$ [16]

[15] The derivation is similar to the case without the government. See, for example, Barro and Sala-i-Martin (2004, ch.3.6).

[16] Now, the NPG condition is $\lim_{t\to\infty} B(t)e^{-(\overline{r}_{ne}(t)-p)t} = 0$.

Note, that the discount rate in the intertemporal budget constraint is lower by the p mortality rate in the case of the government compared to the case of households. The underlying reason is that the time horizons of the two actors differ from each other: contrary to the households, the time horizon of the government continues to be infinite.

According to equation (22), in steady state, the debt-to-GDP ratio is constant at the following value:

$$\mu^* = \frac{\Gamma_B^* / Y^*}{r^* - g - n} \quad . \tag{23}$$

The dynamics of the system is determined by equations (6), (20) and (22).[17] The steady state is described by equations (8), (21) and (23).

Substituting equation (8) for $\hat{c}^*$ and $\mu^* f(\hat{k}^*)$ for $\hat{b}^*$ in equation (21), we arrive at the following alternative expression for the long-run debt-to-GDP ratio after some manipulation:

$$\mu^* = \left[1 - \phi - (n + \delta + g)\frac{\hat{k}^*}{f(\hat{k}^*)}\right]\frac{r_{ne}^* - (g + \rho + p)}{x} - \frac{\hat{k}^*}{f(\hat{k}^*)} \quad . \tag{24}$$

The advantage of equation (24) over equation (23) is that in the former case the steady-state debt-to-GDP ratio is expressed solely as a function of the steady-state physical capital per effective labor, which is of first-order importance from our point of view. Namely, the crucial step with regard to the derivation of the crowding-out effect is the differentiation of the debt-to-GDP ratio (eq.24) with respect to the physical capital per effective labor:[18]

[17] The equation of motion for $\hat{k}$ is the same in the Blanchard model as in the RCK model.

[18] The capital income tax rate is considered to be independent of $\hat{k}^*$ during the differentiation $(\partial \tau_A / \partial \hat{k}^* = 0)$. Strictly speaking, this assumption does not hold since $\tau_A = F(\mu^*,\ f'(\hat{k}^*) - \delta,\ \hat{k}^* / \hat{y}^*)$ according to equation (23). (See equation (A.7) in the appendix for further details.) The underlying reasons are twofold. First, the interest rate on fairly safe government bonds is assumed to be equal to the net marginal product of capital similarly to the rate of return of any other assets on the market.

$$\frac{\partial\mu^*}{\partial\hat{k}^*}=\left[-(n+\delta+g)\frac{f(\hat{k}^*)-\hat{k}^*f'(\hat{k}^*)}{f^2(\hat{k}^*)}\right]\frac{r_{ne}^*-(g+\rho+p)}{x}+$$

$$+\left[1-\phi-(n+\delta+g)\frac{\hat{k}^*}{f(\hat{k}^*)}\right]\frac{(1-\tau_A)f''(\hat{k}^*)}{x}-\frac{f(\hat{k}^*)-\hat{k}^*f'(\hat{k}^*)}{f^2(\hat{k}^*)}<0 \quad . \tag{25}$$

Because the sign of the derivative in equation (25) is negative, an increase in the debt-to-GDP ratio leads to lower steady-state capital and vice versa.[19] Thus, public debt crowds out physical capital and reduces long-run output in the Blanchard model as opposed to the RCK model. This crucial result was already documented in the original paper of Blanchard (1985).

Equation (25) proves not only the existence of crowding out but is also appropriate to determine the magnitude of it. The calculation requires the specification of the production function. Therefore, in what follows, we assume a Cobb-Douglas production function: $\hat{y}=\hat{k}^\alpha$, where $0<\alpha<1$. With this end in view, multiplying equation (25) by $\hat{k}^*$ and taking into account that $f(\hat{k})-\hat{k}f'(\hat{k})=\hat{w}$, and $f''(\hat{k})=-\alpha(1-\alpha)\hat{y}/\hat{k}^2$, we arrive at the basic underlying expression of our calculation exercise:

$$\frac{\partial\mu^*}{\partial\hat{k}^*}\hat{k}^*=-(n+\delta+g)\frac{\hat{w}^*}{\hat{y}^*}\frac{\hat{k}^*}{\hat{y}^*}\frac{\hat{a}^*}{\hat{c}^*}-\frac{(1-\tau_A)\alpha(1-\alpha)}{x}\frac{\hat{c}^*}{\hat{y}^*}\frac{\hat{y}^*}{\hat{k}^*}-\frac{\hat{w}^*}{\hat{y}^*}\frac{\hat{k}^*}{\hat{y}^*}=\frac{1}{\Phi} \quad . \tag{26}$$

Equation (26) shows the percentage point change in the debt-to-GDP ratio due to a one percent change in the capital per effective labor. This is the inverse of the crowding-out effect of public debt (*Φ*).

This is an unrealistic assumption which will be discussed shortly in the appendix. Second, in the case of fixed tax rates, the government revenues do not change proportionally with the GDP because of the concavity of the production function. Despite the above, the $\partial\tau_A/\partial\hat{k}^*=0$ restriction is motivated by our interest in the classical crowding-out effect and disinterest in the effect of distortionary taxation related to public debt.

[19] To prove this, recall that $f(\hat{k}^*)-\hat{k}^*f'(\hat{k}^*)=\hat{w}^*>0$, $\left[r_{ne}^*-(g+\rho+p)\right]/x=\hat{a}^*/\hat{c}^*>0$, $f''(\hat{k}^*)<0$ and $\left[f(\hat{k}^*)-\phi f(\hat{k}^*)-(n+\delta+g)\hat{k}^*\right]\Big/f(\hat{k}^*)=\hat{c}^*/\hat{y}^*>0$.

Considering the latter and the fact that $\Delta\hat{y}/\hat{y} = \alpha\Delta\hat{k}/\hat{k}$ in the case of a Cobb-Douglas production function, we can calculate the burden of public debt as

$$\frac{\partial \hat{y}^* / \hat{y}^*}{\partial \mu^*} = \alpha \frac{\partial \hat{k}^* / \hat{k}^*}{\partial \mu^*} = \alpha \cdot \Phi \quad . \tag{27}$$

Blanchard and Fischer (1989) were the first to derive the closed-form solution on the crowding-out effect in the Blanchard model based on the steady-state consumption and capital equations. However, they worked with lump-sum taxes and determined the $\partial K / \partial B$ marginal effect instead of the percentage effect of the debt-to-GDP ratio on long-run output. Consequently, their related formula is inappropriate to perform easy and direct calculations on the burden of public debt. To our knowledge, we are the first to present equations (24-27).

In what follows, we calculate the long-run output loss of public debt based on real data according to equations (26) and (27). The sample covers the period of the 'great moderation' (2000-2007) and incorporates the 167 countries of the Penn World Table 8.0.[20] To obtain robust results, the calculations are performed for six country groups and countries as well. The individual countries under consideration are the USA, the United Kingdom (UK), Japan (JPN), Germany (GER), Italy (ITA) and France (FRA). The six groups of countries are the OECD countries, the Eurozone (EURO), the low income countries (LIC), the lower middle income countries (LMIC), the upper middle income countries (UMIC) and finally the high income countries (HIC). In the last four cases, the 167 countries are grouped according to the income classification of the World Bank. During the calculations, we use the median values of the individual variables in the investigated period.

[20] We neglected the years of the global crisis because the crowding-out effect cannot prevail in a balance sheet recession (Krugman, 2012). Nevertheless, we also performed the calculations for the years between 2008 and 2011. The results are similar to those in Table 1 and are provided upon request.

Table 1
The crowding-out effect of public debt for different countries and country groups

	LIC	LMIC	UMIC	HIC	OECD	EURO	USA	JPN	UK	GER	ITA	FRA	Source/Calculation
ρ	0.037	0.045	0.067	0.046	0.038	0.036	0.027	0.022	0.061	0.029	0.024	0.023	equation (24)
τ_A	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	calibration
$1/p$	32.8	45.0	52.0	58.0	58.6	58.4	57.2	61.8	58.6	58.5	60.5	59.7	WDI
p	0.030	0.022	0.019	0.017	0.017	0.017	0.017	0.016	0.017	0.017	0.017	0.017	
n	0.027	0.018	0.012	0.006	0.005	0.004	0.010	0.001	0.005	0.000	0.007	0.007	PWT 8.0
g	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	calibration
δ	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	calibration
$\hat{w}^*/\hat{y}^*$	0.617	0.532	0.462	0.561	0.592	0.589	0.650	0.529	0.633	0.638	0.539	0.629	PWT 8.0
$\hat{k}^*/\hat{y}^*$	2.563	3.066	2.973	3.013	3.046	3.156	2.999	4.221	2.251	3.081	4.002	3.347	PWT 8.0
$\hat{b}^*/\hat{y}^*$	0.800	0.614	0.448	0.441	0.477	0.519	0.611	1.751	0.470	0.649	1.055	0.637	HPDD
$\hat{c}^*/\hat{y}^*$	0.742	0.712	0.634	0.561	0.577	0.584	0.728	0.543	0.686	0.600	0.588	0.601	PWT 8.0
$\hat{b}^*/\hat{c}^*$	1.078	0.862	0.707	0.786	0.827	0.889	0.839	3.227	0.686	1.081	1.794	1.059	$(\hat{b}^*/\hat{y}^*)/(\hat{c}^*/\hat{y}^*)$
$\hat{k}^*/\hat{c}^*$	3.452	4.305	4.691	5.370	5.282	5.403	4.118	7.779	3.282	5.131	6.804	5.567	$(\hat{k}^*/\hat{y}^*)/(\hat{c}^*/\hat{y}^*)$
$\hat{a}^*/\hat{c}^*$	4.530	5.168	5.399	6.156	6.109	6.291	4.957	11.006	3.967	6.213	8.599	6.627	$(\hat{b}^*/\hat{c}^*)+(\hat{k}^*/\hat{c}^*)$
x	0.004	0.003	0.003	0.001	0.001	0.001	0.001	0.001	0.002	0.001	0.001	0.001	$(\rho+p)(p+n)$
$\partial f(\hat{k}^*)/\partial\hat{k}^*$	0.149	0.153	0.181	0.146	0.134	0.130	0.117	0.112	0.163	0.117	0.115	0.111	$\alpha(\hat{y}^*/\hat{k}^*)$
$1/\Phi$	-15.523	-18.280	-16.886	-25.961	-29.936	-32.115	-36.859	-40.031	-32.995	-44.921	-32.426	-37.082	equation (26)
Φ	**-0.064**	**-0.055**	**-0.059**	**-0.039**	**-0.033**	**-0.031**	**-0.027**	**-0.025**	**-0.030**	**-0.022**	**-0.031**	**-0.027**	
$\frac{\partial\hat{y}^*/\hat{y}^*}{\partial\mu^*}$	**-0.025**	**-0.026**	**-0.032**	**-0.017**	**-0.014**	**-0.013**	**-0.009**	**-0.012**	**-0.011**	**-0.008**	**-0.014**	**-0.010**	equation (27)

Notes: The data in the table are the medians for the individual panels and time series in the period of 2000-2007. The sources of the data are the WDI (World Development Indicators, downloaded: 24.06.2014), the PWT 8.0 (Penn World Table 8.0, Feenstra et al. (2015)) and the HPDD (Historical Public Debt Database (2013 Fall vintage) - IMF, Abbas et al. (2010)). The subjective discount rate is calculated according to equation (24). The expected lifetime at birth adjusted for the inactive years is *1/p* (expected lifetime-20). The labor share $(\hat{w}^*/\hat{y}^*=1-\alpha)$, the consumption share $(\hat{c}^*/\hat{y}^*)$ and the physical capital per output $(\hat{k}^*/\hat{y}^*)$ are measured by and calculated according to the ‘labsh’, the ‘csh_c’, the ‘rkna’ and the ‘rgdpna’ variables of PWT 8.0.

The inputs and the results of the calculations are presented in Table 1.[21] According to our results, neither the magnitude of the crowding out of physical capital nor the resulting loss in the long-run output are significant. The calculations show that a one percentage point change in the debt-to-GDP ratio reduces the steady-state per capita output only by 0.008-0.032 percent throughout the sample.[22]

The conclusions of the Blanchard model in relation with the crowding-out effect of public debt are reasonable. The presence of the crowding out of physical capital is due to the lack of intergenerational links and the finite horizon of households. Namely, the finite horizon assumption implies that a deficit financed temporary tax cut increases not just the actual income but to some extent the present value of lifetime income as well. Moreover, because of the absence of intergenerational links the future tax burden of the present deficit financing will fall to some extent on new households (generations) from which current households (generations) feel themselves disconnected.[23] The negligible magnitude of the crowding-out effect can also be explained. Although individuals maximize their utility during their lifetimes, the time horizon is very long; a minimum of 30-40 years even if we correct for inactive years.

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[21] Note that the subjective discount rate is treated as an endogenous variable and is calculated according to equation (24). The underlying reason is that if all variables and parameters are considered to be exogenous, that is, they are either measured or calibrated, then equation (24) holds only by accident. Because equation (26) enables the calculation of the crowding-out effect with calibrated discount rate as well and because the burden of public debt is not sensitive to the rho parameter (the related results are provided upon request), the endogeneity of the subjective discount rate is only of mathematical importance.

[22] As a caveat, we must keep in mind that equation (26) pertains to the long run. Therefore, the accuracy of the quantitative analyses depends on the extent to which the parameters and the variables in Table 1 characterize the countries and country groups in the long run. However, this probably does not influence our qualitative conclusions because the magnitude of the crowding-out effect is marginal in each case despite the very different social, economic and institutional backgrounds.

[23] Technically, the reason for the net wealth effect of public debt is the lower discount rate in the intertemporal budget constraint of the government compared to the case of households (Blanchard, 1985). Weil (1989) and Buiter (1988) proved that the failure of Ricardian equivalence in the basic Blanchard model can be traced back to the entrance of new generations which are disconnected from the old ones.

This means that although government bonds (debt) are net wealth for households, as equation (21) suggests, it affects the yearly permanent income marginally at best.

To test the robustness of our results, we perform sensitivity analyses with respect to the capital tax rate, the depreciation rate, the debt-to-GDP ratio and the expected lifetime. Despite the fact that the debt-to-GDP ratio and the expected lifetime are not calibrated parameters, we find it important to perform the sensitivity analyses with respect to these variables as well for different reasons. First, the relationship of government debt with output is non-linear in the Blanchard model because the crowding-out effect depends on the initial debt-to-GDP ratio (eq.26). Moreover, non-linearity and threshold effects are heavily investigated empirical issues in the recent debt-growth literature. Second, "If we think of 1/p as the horizon index, we can choose it anywhere between zero and infinity and study the effects of the horizon of agents on the behavior of the economy" (Blanchard, 1985 pp.224.).[24]

The underlying country group of the sensitivity analyses is the Eurozone. The results are presented in Figure 2.[25] These results show that the crowding-out effect is robust to the debt-to-GDP ratio and the tax rate, whereas it is moderately sensitive to the depreciation rate and the horizon index in the Blanchard model. However, in the latter cases, the magnitude of the crowding-out effect continues to remain negligible throughout the entire interval of the depreciation rate and throughout the reasonable

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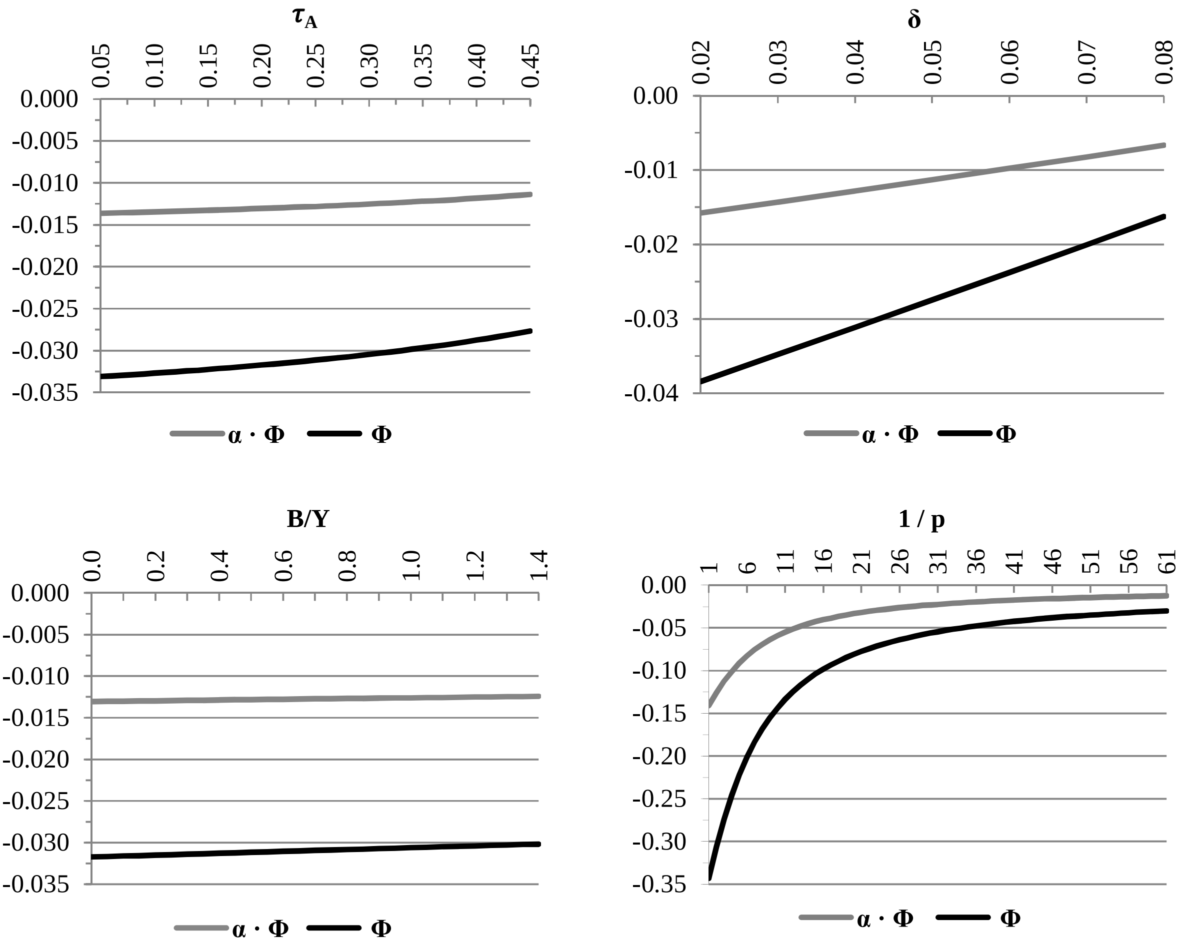
[24] However, we have to emphasize that the treatment of *1/p* as a horizon index is only a theoretical experiment. Namely, it is not possible to separate the expected lifetime from the time horizon of the agents in the frame of the Blanchard model. The reason is that it is implausible that an agent pursues sequential optimization during her life and consumes all of her wealth until the end of the individual sub-periods every time.

[25] The sensitivity with respect to the technology growth rate was also investigated. The results are not published because of lack of space. However, they show that the crowding-out effect is robust to g.

interval (i.e., $1/p > 10\text{-}15$) of the horizon index.[26] Consequently, our former conclusions depend neither on the parameter calibration nor on the measured debt-to-GDP ratios and expected lifetimes.

Figure 2. The sensitivity of the crowding-out effect and the resulting output loss in the Eurozone



Notes: The calculations are based on equations (26) and (27). The variables and the parameters take on the values of the Eurozone in each case (Table 1).

[26] In the case of a shorter horizon index (i.e., $1/p < 10$), the crowding-out effect and the related output loss are somewhat larger, but they are still not very impressive.

To illustrate the negligible magnitude of the crowding-out effect in the Blanchard model, we calculate the total burden of public debt at a 90 percent debt-to-GDP ratio, corresponding to the euro area average. That is, we estimate how much higher steady-state output would be without public debt. Because the crowding-out effect is not sensitive to *B/Y*, the total impact on output can be calculated by multiplying the marginal burden of debt ($\alpha\Phi$) by the debt-to-GDP ratio: $\Delta\hat{y}^* / \hat{y}^* = 0.9 \cdot (-0.032) = -0.0288$.[27] According to this simple calculation, we can conclude that the rise of the debt-to-GDP ratio from zero to 90 percent reduces the long-run output – via the classical crowding out channel – by approximately 2-3 percent. This is not a considerable loss.

The distortionary taxation related to public debt does not change the picture. As the appendix demonstrates, contrary to the RCK model, it is impossible to calculate analytically the output loss triggered by distortionary taxes in the Blanchard model. Nevertheless, we can suppose that the magnitudes are similar in the two models. Thus, taking into account the results of the appendix on the RCK model, we can conclude that, adding the effect of distortionary taxes to the classical crowding-out effect, the total output loss of a 90 percent debt-to-GDP ratio probably does not exceed 5-6 percent in the frame of the Blanchard model. Consequently, the burden of public debt is not a serious concern in the basic Blanchard model either.

Our conclusion is completely in line with that of Evans (1991) who was the first to prove that Ricardian equivalence is a good approximation in the Blanchard model. His numerical results are of similar magnitude to ours. Nevertheless, the approximate neutrality of fiscal policy in the Blanchard model can hinge crucially upon the absence of some relevant life-cycle aspects of households' behavior. In fact, the basic Blanchard model with age-independent mortality rate and wages can be considered rather as a model of dynasties with finite horizon than as a classical OLG model. Faruqee et al. (1997) and Faruqee and Laxton (2000) show that if wages follow a hump-shaped life-cycle pattern than the burden of public debt can be considerable in the Blanchard model as well. In another paper,

[27] To provide a pessimistic estimation, we use the largest marginal impact on output in Table 1.

Faruqee (2003) arrives at the same conclusion by introducing a death probability which increases with the age. According to these papers the long run output loss of a 90 percent debt-to-GDP ratio can be posited between 5 and 10 percent. However, the exact value is very sensitive to the parameter calibration.[28] So, we decided not to depart form the basic Blanchard framework. Moreover, in the next section, the Solow model also delivers output losses in the 5-10 percentage range for developed countries at the 90 percent debt-to-GDP ratio. Consequently, the main policy conclusion of the paper with regard to the – upper bound of the – burden of public debt is unaffected by these potential modifications of the basic Blanchard framework.

4. Government debt in the Solow model with human capital

The dynamic optimization of households and the underlying permanent income hypothesis can be heavily challenged on both theoretical and empirical grounds (Romer, 2012). The central predictions of the permanent income hypothesis are that households base their consumption decision on the present value of their future lifetime income and that consumption growth is determined by the difference between the interest rate and the subjective discount rate. In other words, the permanent income hypothesis predicts the consumption path of households to be practically independent of their current income. However, many empirical studies find a strong positive correlation between current income and consumption (e.g., Caroll and Summers, 1991). Furthermore, some studies find that predictable changes in income lead to predictable changes in consumption at the time that they happen, which also contradicts the permanent income hypothesis (Shapiro and Slemrod, 2003; Johnson et al., 2006).

Because of the objections raised against the permanent income hypothesis, we drop the assumption of dynamically optimizing households in this section and switch to the traditional consumption theory, which assumes that households base their consumption on their current disposable income and follow a

[28] For example, in Faruqee and Laxton (2000), the burden of public debt decreases with the intertemporal elasticity of substitution and becomes similar to the results of the basic Blanchard model as log-utility is reached.

rule-of-thumb decision. This implies practically that we study the impact of public debt on steady-state output in the framework of the Solow model with a constant and exogenous household saving rate. The constant, exogenous saving rate of households implies that government deficit reduces aggregate savings – and thereby investments as well – one-to-one in the extreme. Therefore, in this section, we consider the case of a complete crowding-out effect in the tradition of Elmendorf and Mankiw (1999).[29]

In addition to the underlying consumption behavior, a further departure from the RCK and the Blanchard model is that we take human capital into account. It is well known that the human capital augmented version of the Solow model fits the empirical observations much better than the basic one (Mankiw et al., 1992). However, our primary reason for considering human capital is that the presence of human capital affects the burden of public debt in a quantitatively important way in the Solow model.[30]

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[29] In fact, complete crowding out is of theoretical importance only, even in the framework of the Solow model. The reason is that government deficit always increases the disposable income of households and thereby to some extent their savings as well. Consequently, government deficit reduces the aggregate saving rate less than one-to-one. For simplicity, we neglect this problem.

[30] The inclusion of human capital into the RCK model and into the Blanchard model would have complicated the discussion of the previous sections considerably, without improving our understanding of the issue. First, the net wealth effect of public debt continues to be zero in the RCK model with human capital as well. (The proof is available upon request.)

Second, the inclusion of human capital into the Blanchard model would affect the burden of public debt marginally at best. To verify this claim, consider the typical adjustment dynamics of an economy in which human capital becomes relatively abundant due to the initial crowding out of physical capital by a rise in public debt (Barro and Sala-i-Martin, 2004, ch.5). The relative abundance of human capital depresses its rate of return compared to the rate of return of physical capital. As a result, households reallocate their savings and invest only into physical capital until the equilibrium ratio of the two types of capital is reached again and the rates of returns become equal. At the end of the adjustment, the higher level of public debt results in lower long-run levels of both capital types. However, the imbalance between the two types of capital induces such a mechanism that counterbalances the initial crowding out of physical capital to some extent. Of course, the output loss caused by public debt certainly differs between the baseline and the augmented model, but the above considerations and the

The discussion is based on the human capital augmented Solow model of Mankiw et al. (1992). The production function is

$$Y = K^{\alpha} H^{\beta} (EL)^{1-\alpha-\beta} \quad ,$$

where H is human capital, $\alpha, \beta > 0$ and $\alpha + \beta < 1$. The steady-state output evolves according to the following well-known expression:

$$\ln(\hat{y}^*) = \frac{\alpha}{1-\alpha-\beta}\ln(s_K^*) + \frac{\beta}{1-\alpha-\beta}\ln(s_H) - \frac{\alpha+\beta}{1-\alpha-\beta}\ln(n+g+\delta) \quad , \tag{28}$$

where s_K and s_H are the saving (investment) rates in relation with physical and human capital respectively.

We calculate the burden of public debt according to the stock approach of Dedák and Dombi (2016). These authors derive a closed form solution on the effect of public debt on long-run output in the frame of the Solow model. The main steps of their derivation are introduced below.

The first step is to calculate the effect of the budget balance on the aggregate saving rate. The aggregate savings is the sum of private and government savings. For simplicity, it is assumed that government savings influences only the accumulation of physical capital:

$$s_K = s_{KP} + \eta \quad , \tag{29}$$

where s_{KP} is the savings of the private sector compared to the output. As we assume complete crowding out, private savings is unaffected by government savings. The steady-state balance of budget can be given as a constant z fraction of private savings: $\eta^* = z \cdot s_{KP}$. Parallel with the latter, equation (29) can be reformulated as $s_K^* = (1+z)s_{KP}$ for the long run. Substituting this new formula for s_K^* in equation (28) and using the fact that $\ln(1+z) \approx z$ if z is close to zero, the long-run output can be rewritten as follows:

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tiny magnitudes of the crowding-out effect calculated in section 3 suggest that the inclusion of human capital into the Blanchard model is not an important issue from a quantitative point of view.

$$\ln(\hat{y}^*) = \Psi_1 \ln(s_{KP}) + \Psi_1 \frac{1}{s_{KP}} \eta^* + \Psi_2 \ln(s_H) + \Psi_3 \ln(n + g + \delta) \quad , \tag{30}$$

where $\Psi_1 = \frac{\alpha}{1-\alpha-\beta}$, $\Psi_2 = \frac{\beta}{1-\alpha-\beta}$ and $\Psi_3 = -\frac{\alpha+\beta}{1-\alpha-\beta}$.

Equation (30) establishes the connection between the balance of budget and the steady-state output. To turn this connection into a connection between public debt and output, we must consider the steady-state debt-to-GDP ratio expressed in equation (9b). Substituting equation (9b) for η^* in equation (30), we arrive at the formula of Dedák and Dombi (2016) which establishes the link between long-run output and public debt:

$$\ln(\hat{y}^*) = \Psi_1 \ln(s_{KP}) - \Psi_4 \mu^* + \Psi_2 \ln(s_H) + \Psi_3 \ln(n + g + \delta) \quad , \quad \text{where } \Psi_4 = \Psi_1 \frac{g+n}{s_{KP}}. \tag{31}$$

The most important message of equation (31) is that the coefficient of μ^*, which represents the marginal effect of public debt on long-run output, is not constant but changes with the private sector's saving rate, the population growth rate, the technology growth rate and the parameters of the production function.[31] Note that the coefficient is negative, so public debt reduces long-run output: the crowding-out effect is at work. The magnitude of output loss emanating from public debt decreases with the private sector's saving rate and increases with the population growth rate. In other words, a high (private) saving rate and low population growth rate result not just in high steady-state output but in a low burden of public debt as well. The intuitive explanation is simple. If the saving rate of the private sector is high, then the budget deficit necessary to maintain a given debt-to-GDP ratio decreases

[31] We emphasize at this point that the inclusion of human capital into the Solow model is quantitatively an important step. If human capital is absent, $\beta=0$ and so Ψ_1 is lower implying thereby a smaller debt coefficient in equation (31) in absolute value. Under standard parameter calibration the difference can be considerable. For example, if $\alpha = \beta = 1/3$, the burden of public debt is twice as high with human capital as without human capital.

the aggregate saving rate only modestly in percentage terms and hence leads to small changes in steady-state output. Furthermore, a higher population growth rate allows a higher government deficit for a given debt-to-GDP ratio (eq.9b), thereby reducing the aggregate saving rate and the output.

In the following, we calculate the marginal effect of the debt-to-GDP ratio on the long-run output ($-\psi_4$) and the total burden of public debt at a 90 percent debt-to-GDP ratio according to equation (31). Figures 3 and 4 present the results as a function of the private sector's saving rate and the population growth rate, respectively. The calibration of the remaining parameters follows the standard assumptions in the literature and is depicted in the figures (Mankiw et al., 1992; Mankiw, 1995).

The marginal effect of public debt and the output loss at a 90% debt-to-GDP ratio....

Figure 3. …as a function of the private saving rate

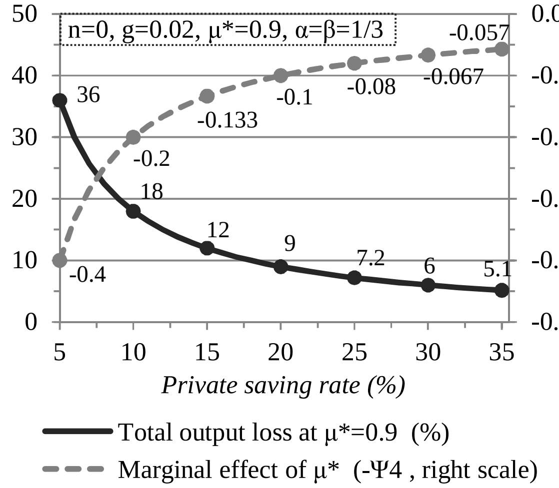
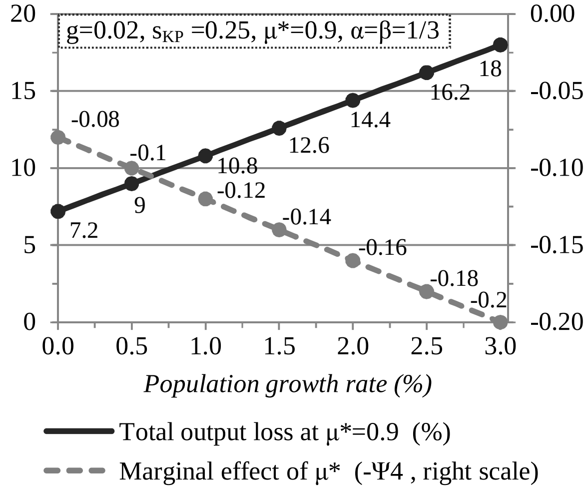


Figure 4. …as a function of the population growth rate



Notes: The calculations are based on equation (31). The numerical results are rounded to three decimal places for $-\psi_4$ and to one decimal place for the total loss in steady-state output $(-100 \cdot 0.9 \cdot (-\psi_4))$.

As Figure 3 shows, the marginal effect of public debt decreases (in absolute value) with the saving rate of the private sector and so does the burden of public debt as well. For example, in the case of a 15 percent private saving rate, $-\psi_4 = -0.133$, which means that a one percentage point increase in the debt-to-GDP ratio reduces the long-run output by 0.133 percent. In this case, the total loss of output emanating from the 90 percent debt-to-GDP ratio is 12 percent. In contrast, at higher saving rates, the output loss is much smaller: for example, at $s_{KP} = 0.25$, the output loss amounts only to 7.2 percent.

The results in Figure 4 show that the marginal effect of public debt increases (in absolute value) with the population growth rate and so does the burden of public debt as well. For example, assuming that the private saving rate is 25 percent, the $-\psi_4$ coefficient is -0.16 at $n = 0.02$, which means that a one percentage point increase in the debt-to-GDP ratio reduces the long-run output by 0.16 percent. In this case, the total loss of output emanating from a 90 percent debt-to-GDP ratio is 14.4 percent. In contrast, at lower population growth rates, these values are much lower.

Note that Figures 3 and 4 are partial analyses because they only investigate the effect of one variable on the burden of public debt. However, in reality, the ceteris paribus condition does not hold. It is well known that the saving rate correlates positively with output, whereas the population growth rate correlates negatively with output (Durlauf et al., 2005). Consequently, the least developed countries are usually characterized by low saving rates and high population growth rates. This implies that the burden of public debt is under double pressure in these countries and can be significantly higher than Figures 3 and 4 might suggest. For example, if the private saving rate is only 15 percent, while the population growth rate is 2 percent, the coefficient of public debt is -0.267 in equation (31). With such a high marginal effect of debt, the total output loss at a 90 percent debt-to-GDP ratio is 24 percent, which is enormously high.

To sum up, we can conclude that in the Solow model with a constant saving rate of households and with complete crowding out of physical capital, the burden of public debt can be remarkably different across countries. In developed countries with high saving rates and low population growth rates, public debt has only a minor impact on the steady-state output. In these countries, it does not matter significantly whether the debt-to-GDP ratio stabilizes at a relatively low or high level in the long run.

In the least developed countries, which are typically characterized by low private saving rates and high population growth rates, the burden of public debt is vast and reducing government indebtedness can be accompanied by a significant rise in steady-state output.

5. Conclusions

This paper investigated the impact of public debt on capital accumulation and long-run output in the framework of neoclassical growth models. From the viewpoint of economic policy, the relevance of the issue is that economists and policy makers advocating fiscal austerity even in times of severe recession usually base their arguments on the long-run burden of public debt.

Regarding the effect of fiscal policy in growth models with full employment, the principal question is how the different methods of financing government expenditures affect the consumption decisions of households. The more the permanent income hypothesis holds, or in other words, the more properly dynamic optimization describes consumer behavior, and the stronger the intergenerational links are, the less public debt affects long-run output. Because the empirics do not support any consumption theory uniformly, we considered three different cases: two extremes and an intermediate one.

The RCK model can be viewed as one of the extreme cases in which we examined the effect of public debt. This model is marked by intergenerational links and by households optimizing on an infinite horizon. Thus, the conditions for Ricardian equivalence to hold are given. In spite of this, if taxes are distortionary, the burden of public debt becomes a relevant issue in the RCK model as well. However, our results show that the tax distortion does not alter the main conclusion of the model and the burden of public debt remains negligible.

The Blanchard model represents the intermediate case in which no intergenerational links are present but consumption behavior is still governed by dynamic optimization. As a result, the time horizon of households' behavior switches from infinite to finite. Because of the lack of intergenerational links, public debt affects the consumption of households directly and results in lower steady-state physical capital and output. However, according to our calculations, the magnitudes of the crowding-out effect and the related output loss are negligible. For example, the increase in the debt-to-GDP ratio from zero to 90 percent lowers the steady-state output only by 2-3 percent via the classical crowding out channel. Taking the effect of distortionary taxes also into account, the picture does not change: the burden of public debt of a 90 percent debt-to-GDP ratio probably remains under 5-6 percent in this case too.

The other extreme case of consumer behavior is presented by the Solow model in which households save a constant fraction of their income as a rule-of-thumb. Therefore, dynamic optimization and intergenerational links are both absent. Under such conditions, the government's budget deficit lowers the aggregate saving rate one-to-one in the extreme; that is, the crowding-out effect of public debt is complete. This implies that the Solow model serves as a means to calculate the upper bound of the burden of public debt. Our main results are that the burden of public debt differs across countries and can be large depending on the saving rate and the population growth rate. In countries with a high saving rate in the private sector and with a low population growth rate, conditions typical for developed countries, the impact of public debt on steady-state output is modest. For example, under standard parameter calibration, if the private sector's saving rate is 25 percent and the population growth is zero, the increase in the debt-to-GDP ratio from zero to 90 percent lowers the steady-state output only by 7.2 percent. In contrast, the burden of public debt is much higher under the conditions of the least developed countries with low saving rates and high population growth rates.

The findings of the Solow model have important implications for the empirical literature as well. They can serve as possible theoretical basis for the latest empirical results, according to which the effect of public debt on economic growth is country-specific and the output losses are much smaller in developed countries than in developing ones.

Our results convey a very important message for European economic policies: the fear of austerity advocates is not supported by neoclassical growth theory, and high public debt is not a serious growth burden in developed countries in the long run. This statement is underpinned further by the fact that the impact of public debt accumulated during a severe crisis differs crucially from that accumulated at potential output. Considering the dire state of European economies in the last few years, our results lead to the conclusion that the switch in the stance of European crisis management toward fiscal austerity in 2010 was probably a fundamental mistake. Instead of insisting on the overall fight against public debt, it would have been more fruitful to continue the stimulation of economic growth by fiscal expansion in those countries that had the needed fiscal space. This policy is still an option.

Appendix: The effect of public debt on steady-state output through distortionary taxes in the RCK model

In order to quantify the effect of public debt on long-run output through distortionary taxation we use the framework of the RCK model presented in section 2. The differences compared to section 2 are the followings. First, the tax rates of the wage income and the capital income are set to be equal for mathematical convenience. Second, the production function takes the Cobb-Douglas form. Considering the above, the steady state and the equilibrium of the economy are described partly by the following equations (asterisks are neglected):

$$\hat{y} = (\hat{k})^{\alpha} \quad , \tag{A.1}$$

$$(1-\tau)r = \rho + g \quad \rightarrow \quad r = \frac{\rho+g}{(1-\tau)} \quad , \tag{A.2}$$

$$r = \alpha\frac{\hat{y}}{\hat{k}} - \delta \quad \rightarrow \quad \frac{\hat{y}}{\hat{k}} = \frac{r+\delta}{\alpha} \quad , \tag{A.3}$$

$$\mu = \frac{(\tau rB + \tau W + \tau rK - G)/Y}{r-(g+n)} = \frac{\tau r\mu + \tau - \tau\delta\hat{k}/\hat{y} - \phi}{r-(g+n)} \quad . \tag{A.4}$$

Equations (A.2), (A.3) and (A.4) are equivalent to equations (7), (3) and (9a) respectively. The last term in equation (A.4) takes into account that $Y = W + rK + \delta K$.

Substituting equation (A.2) and equation (A.3) for r and $\hat{k}/\hat{y}$ in equation (A.4), we arrive at a quadratic equation in the tax rate after some manipulation:

$$(\delta-\delta\alpha)\tau^2 + ((\alpha-\mu\rho+\mu n-\phi-1)\delta-\rho-g)\tau + (\mu\rho-\mu n+\phi+\mu g+\mu\delta)\rho + (g+\delta)(\phi-\mu n) = 0 \quad . \tag{A.5}$$

The solution of equation (A.5) delivers the tax rate which is consistent with the targeted μ debt-to-GDP ratio. After the determination of the tax rate, the calculations of the steady-state interest rate, capital-output ratio and output per effective labor are straightforward, according to equations (A.1), (A.2) and

(A.3). Table A.1 presents the results of the calculations for different debt-to-GDP ratios based on the typical parameter settings for developed countries.[32]

As it can be seen in Table A.1, the tax rate increases with the long-run debt-to-GDP ratio. The underlying reason is the need to achieve a higher primary budget surplus in order to maintain a higher debt-to-GDP ratio. The increment in the tax rate compared to the $\mu = 0$ case can be interpreted as the tax burden of public debt. The consequences of the higher tax rate are the higher interest rate and thus the lower capital and output at the same time.

Although public debt reduces the long-run output due to tax distortions, our results show that this effect is not important from a quantitative point of view. For example, if the long-run debt-to-GDP ratio jumps from 0 to 90 percent, the tax rate and the interest rate increase only by 2.8 and 0.7 percentage points respectively. In accordance with these rather small movements, the steady-state output decreases only by 2.75 percent.

We performed an extended robustness analysis and found that the output loss of public debt triggered by distortionary taxation is sensitive to the n, α, ϕ and ρ parameters. The results show that the output loss decreases with the population growth rate and increases with the subjective discount rate, the capital share and the government expenditures relative to GDP. However, as Table A.2 demonstrates, the magnitudes remain small: the tax burden of a 90 percent debt-to-GDP ratio depresses the long-run output less than 4-5 percent in almost each case. Beside this general observation, Table A.2 reveals another tendency. The output loss is larger for those parameter combinations which are more typical for less developed countries (gray-colored fields). Nevertheless, the advantage of developed countries with regard to the burden of public debt is not as dramatic in percentage point terms as in the Solow model.

[32] The ϕ parameter is calibrated in accordance with the average 'total government expenditure net of interest payment' for the Eurozone, which was 46.7 percent in 2013 (Eurostat). The tax rate corresponds to the smaller root of equation (A.5), because the larger root is above one in each case.

Table A.1 The effect of public debt on steady-state output through distortionary taxes

$n = 0,\ \alpha = 0.4,\ \delta = 0.04,\ g = 0.02,\ \phi = 0.45,\ \rho = 0.03$											
μ^* (%)	**0**	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**
τ^* (%)	50.7	51	51.4	51.7	52	52.3	52.6	52.9	53.2	53.5	53.8
r^* (%)	10.1	10.2	10.3	10.3	10.4	10.5	10.5	10.6	10.7	10.8	10.8
$\hat{k}^* / \hat{y}^*$	2.827	2.814	2.801	2.789	2.776	2.763	2.750	2.737	2.724	2.711	2.698
$\hat{y}^*$	1.999	1.993	1.987	1.981	1.975	1.969	1.963	1.957	1.951	1.944	1.938
Tax burden of public debt (ppt)	0.0	0.3	0.7	1	1.3	1.6	1.9	2.2	2.5	2.8	3.1
Output loss (%)	**0.00**	**0.30**	**0.60**	**0.91**	**1.21**	**1.51**	**1.82**	**2.13**	**2.44**	**2.75**	**3.06**

Notes: 'ppt' stands for percentage points. The tax burden is calculated as follows: $(\tau^* - \tau^*_{\mu=0})$. The output loss is calculated as follows: $-100(\hat{y}^* - \hat{y}^*_{\mu=0}) / \hat{y}^*_{\mu=0}$

Table A.2 The sensitivity of the output loss related to the tax burden of μ* = 90 (%)

Partial Sensitivity	n		α			ϕ			ρ		
	0.01	0.02	0.35	0.45	0.50	0.35	0.40	0.50	0.02	0.04	0.05
Output loss (%)	1.82	0.91	2.18	3.42	4.24	2.23	2.46	3.09	1.76	3.77	4.82

Joint Sensitivity	α=0.35 ρ=0.02	α=0.45 ρ=0.05	n=0.01 ρ=0.02	n=0.02 ρ=0.05	ϕ=0.5 ρ=0.02	ϕ=0.35 ρ=0.05	ϕ=0.5 ρ=0.02 α=0.35 n=0.01	ϕ=0.35 ρ=0.05 α=0.45 n=0.02
Output loss (%)	1.39	5.97	0.88	2.86	1.99	3.95	0.78	2.92

Notes: The results of the joint sensitivity analysis are colored gray if the underlying parameter combination is more typical for less developed countries than for developed ones. In the opposite case, the fields are left uncolored.

In section 2, we worked with the basic RCK model, which ignored important elements of the economy. For example, human capital and consumption tax were not included into the model. Although human capital and consumption tax do not influence the Ricardian equivalence in the RCK model, they do affect the output loss resulting from distortionary taxes. If human capital is present, the loss in long-run output is higher, because the decrease of the physical capital in the wake of a tax increase leads to lower human capital as well.[33] On the other hand, the inclusion of the consumption tax reduces the distortionary effect of taxation related to public debt, because the larger tax base, that is the larger set of taxable transactions and incomes, renders a lower tax rate on the return of physical capital possible.

Moreover, the tax burden of public debt decreases substantially compared to the textbook case if we introduce uncertainty and thereby depart from the assumption that the interest rate on government bonds equals to the marginal product of capital. The underlying reason is that the interest rate on the fairly safe government bonds is considerably lower than the expected rates of return on the more risky market assets.

In order to assess whether our conclusions with regard to the distortionary taxation stemming from public debt and the related output loss are relevant under more realistic conditions as well, we performed the calculation for the case when human capital, consumption tax and the difference between the rates of return on government bond and market assets are taken into account in the RCK model.[34] Our results show that the output loss rises with the interest rate on government bonds, but remains small under the conditions of developed countries. For example, the tax burden of a 90 percent debt-to-GDP ratio decreases the long-run output only by 2.7 percent if the real interest rate on government bonds is set to 4 percent. The output loss rises to 4.5 percent if the real interest rate on government bonds increases to 5 percent. These numbers can be considered as upper bounds for developed countries because it is not realistic that the real interest rate paid on their government bonds permanently exceeds 4-5 percent. In contrast, the output loss of the tax burden can grow large in less

[33] For the RCK model augmented with human capital a la Mankiw et al (1992), see Acemoglu (2009, ch.10.4).

[34] The calculation followed the same line as in the baseline case and was based on the human capital and consumption tax augmented version of the RCK model with fixed government bond rate. Those parameters which were already present in the basic RCK model were calibrated as in Table A.1. The new parameters, that is the share of human capital in the production (β), the tax rate on consumption (τ_C) and the tax rate on the interest income from government bonds (τ_B) were set as follows: $\beta = 0.25$, $\tau_C = \tau_B = 0.2$. The details of the calculation and the results are provided upon request.

developed countries with considerably higher bond rates. For example, if the real interest rate on government bonds falls into the range of 8-10 percent, the tax burden of a 90 percent debt-to-GDP ratio reduces long-run output by 10-13 percent. Of course the numerical results vary with the parameter calibration to some extent, but our main conclusion holds in the augmented RCK model as well: the burden of public debt brought about by distortionary taxes is not an important issue in developed countries.

Finally, we have to discuss shortly the case of the Blanchard model. The equation for the steady-state debt-to-GDP ratio (eq.23) can be rewritten as follows by taking into account that $Y = W + rK + \delta K$ (asterisks are neglected again):

$$\mu = \frac{\tau(r+p)\mu + \tau - \tau\delta\hat{k}/\hat{y} + \tau p\hat{k}/\hat{y} - \phi}{r-n-g} \quad . \tag{A.6}$$

Rearranging equation (A.6), we arrive at the tax rate expressed as a function of the targeted debt-to-GDP ratio, the interest rate and the capital-to-output ratio:

$$\tau = \frac{\mu(r-n-g)+\phi}{(r+p)\mu + 1 - \delta\hat{k}/\hat{y} + p\hat{k}/\hat{y}} \quad . \tag{A.7}$$

To derive the tax burden of public debt, we have to substitute for r and $\hat{k}/\hat{y}$ in equation (A.7) so as to express the tax rate solely as a function of μ and the model parameters. Since the Blanchard model assumes competitive markets, equation (A.3) can be used to substitute for $\hat{k}/\hat{y}$. However, contrary to the RCK model, the interest rate cannot be expressed as the function of the model parameters and the tax rate. The reason is that the equation of motion for $\hat{c}$ (eq.20) contains other endogenous variables as well beside r and τ. This lack of counterpart of equation (A.2) implies that the tax burden of public debt cannot be calculated analytically in the Blanchard model.

References

Abbas, S.A., Belhocine, N., ElGanainy, A., Horton, M., 2010. A historical public debt database. *IMF Working Paper* no.WP/10/245, International Monetary Fund.

Acemoglu, D., 2009. *Introduction to Modern Economic Growth.* Princeton University Press, Princeton, New Jersey.

Alesina, A., Ardagna, S., 2010. *Large Changes in Fiscal Policy: Taxes versus Spending.* In: Brown, J.R. (Ed.), Tax Policy and the Economy, vol. 24. The National Bureau of Economic Research, University of Chicago Press, pp. 35-68.

Auerbach, A.J., Kotlikoff, L.J., 1987. *Dynamic Fiscal Policy*. Cambridge University Press,

Ball, L., Mankiw,N.G., 1995. What do budget deficits do? *NBER Working Paper* no.5263, National Bureau of Economic Research.

Barro, R.J., 1974. Are government bonds net wealth? *Journal of Political Economy* 82(6), 1095-1117.

Barro, R.J., 1976. Perceived wealth in bonds and social security and the Ricardian equivalence theorem: Reply to Feldstein and Buchanan. *Journal of Political Economy* 84(2), 343-349.

Barro, R.J., 1990. Government spending in a simple model of endogenous growth. Journal of Political *Economy* 98(5), S103-S125.

Barro, R.J., Sala-i-Martin, X., 2004. *Economic Growth*. (2nd ed.) The MIT Press, Cambridge, Massachusetts.

Bernheim, B.D., 1987. *Ricardian Equivalence: An Evaluation of Theory and Evidence*. In: Fischer, S. (Ed.), NBER Macroeconomics Annual, vol. 2. The National Bureau of Economic Research, The MIT Press, pp. 263-304.

Blanchard, O.J., 1985. Debt, deficits, and finite horizons. Journal of Political Economy 93 (2), 223-247.

Blanchard, O.J., Fischer, S., 1989. *Lectures on Macroeconomics*. The MIT Press, Cambridge, Massachusetts.

Blanchard, O.J., Leigh, D., 2013. Growth forecast errors and fiscal multipliers. *The American Economic Review* 103(3), 117-120.

Broner, F., Erce, A., Martin, A., Ventura, J., 2014. Sovereign debt markets in turbulent times: Creditor discrimination and crowding-out effects. *Journal of Monetary Economics* 61(1), 114–142.

Buiter, W.H., 1988. Death, birth, productivity growth and debt neutrality. *The Economic Journal* 98(391), 279-293.

Caroll, C.D., Summers, L.H., 1991. *Consumption Growth Parallels Income Growth: Some New Evidence*. In: Bernheim, B.D., Shoven, J.B. (Eds.), National Saving and Economic Performance, The National Bureau of Economic Research, University of Chicago Press, pp. 305-348.

Cass, D., 1965. Optimum growth in an aggregative model of capital accumulation. *The Review of Economic Studies* 32(3), 233-240.

Checherita-Westphal, C., Rother, P., 2012. The impact of high government debt on economic growth and its channels: An empirical investigation for the euro area. *European Economic Review* 56 (7), 1392-1405.

Dedák, I., Dombi, Á., 2016. Public debt in the Solow model: the end of the debt fairy. *Working Paper* (Available at SSRN: http://ssrn.com/abstract=2746805)

DeLong, J.B., Summers, L.H., 2012. Fiscal policy in a depressed economy. *Brookings Papers on Economic Activity* (Spring), 233-274.

Diamond, P.A., 1965. National debt in a neoclassical growth model. *The American Economic Review* 55(5), 1126-1150.

Dreger, C., Reimers, H.-E., 2013. Does euro area membership affect the relation between GDP growth and public debt? *Journal of Macroeconomics* 38(Part B) 481-486.

Durlauf, S.N., Jonhson, P.A., Temple, J.R.W., 2005. *Growth Econometrics*. In: Aghion, P., Durlauf, S.N. (Eds.), Handbook of Economic Growth, vol. 1A. Elsevier, Amsterdam, pp.555-677.

Égert, B., 2015. Public debt, economic growth and nonlinear effects: Myth or reality? *Journal of Macroeconomics* 43 (March), 226–238.

Eberhardt, M., Presbitero, A.F., 2015. Public debt and growth: Heterogeneity and non-linearity. *Journal of International Economics* 97(1) 45-58.

Eggertsson, G.B., Krugman, P., 2012. Debt, deleveraging, and the liquidity trap: A Fisher-Minsky-Koo approach. *The Quarterly Journal of Economics* 127(3), 1469-1513.

Elmendorf, D.W., Mankiw, N.G., 1999. *Government Debt*. In: Taylor, J.B., Woodford, M. (Eds.), Handbook of Macroeconomics, vol. 1C. Elsevier, Amsterdam, pp. 1615-1669.

Evans, P., 1991. Is Ricardian equivalence a good approximation? *Economic Inquiry* 29(4), 626-644.

Faruqee, H., 2003. Debt, deficits, and age-specific mortality. *Review of Economic Dynamics* 6(2), 300–312.

Faruqee, H., Laxton, D., 2000. Life-cycles, dynasties, saving: Implications for closed and small, open economies. *IMF Working Paper* no.WP/00/126, International Monetary Fund.

Faruqee, H., Laxton, D., Symansky, S., 1997. Government debt, life-cycle income, and liquidity constraints: Beyond approximate Ricardian equivalence. *Staff Papers (International Monetary Fund)* 44(3), 374-382.

Feenstra, R.C., Inklaar, R., Timmer, M.P., 2015. The next generation of the Penn World Table. The *American Economic Review* 105(10), 3150-3182.

Ghosh, A.R., Kim, J.I., Mendoza, E.G., Ostry, J.D., Qureshi, M.S., 2013. Fiscal fatigue, fiscal space and debt sustainability in advanced economies. *The Economic Journal* 123(566), F4-F30.

Greiner, A., 2008. Human capital formation, public debt and economic growth. *Journal of Macroeconomics* 30(1), 415–427.

Greiner, A., 2011. Economic growth, public debt and welfare: Comparing three budgetary rules. *German Economic Review* 12(2), 205-222.

Greiner, A., 2012. Public debt in a basic endogenous growth model. *Economic Modelling* 29(4), 1344–1348.

Greiner, A., Fincke, B., 2009. *Public Debt and Economic Growth*. Springer Verlag, Berlin, Heidelberg.

IMF, 2014. World Economic Outlook: Legacies, Clouds, Uncertainties. International Monetary Fund (October).

IMF, 2015. World Economic Outlook: Uneven Growth, Short- and Long-Term Factors. International Monetary Fund (April).

Johnson, D.S., Parker, J.A., Souleles, N.S., 2006. Household expenditure and the income tax rebates of 2001. *The American Economic Review* 96 (5), 1589-1610.

Koopmans, T.C., 1965. On the Concept of Optimal Economic Growth. In: The Econometric Approach to Development Planning. *Pontifica Academia Scientarium Scripta Varia* 28 (Part 1), pp. 225-287.

Kourtellos, A., Stengos, T., Tan, C.M., 2013. The effect of public debt on growth in multiple regimes. *Journal of Macroeconomics* 38(4), 35-43.

Krugman, P., 2012. *End This Depression Now!* W.W. Norton & Company, New York, London.

Lane, P.R., 2012. The european sovereign debt crisis. *Journal of Economic Perspectives* 26(3), 49-68.

Mankiw, N.G., Romer, D., Weil, D.N., 1992. A contribution to the empirics of economic growth. *The Quarterly Journal of Economics* 107(2), 407-437.

Mankiw, N.G., 1995. The growth of nations. *Brookings Papers on Economic Activity* (1), 275-326.

Mankiw, N.G., 2000. The savers-spenders theory of fiscal policy. *The American Economic Review* 90(2), 120-125.

Ramsey, F.P., 1928. A mathematical theory of saving. *The Economic Journal* 38(152), 543-559.

Reinhart, C.M., Rogoff, K.S., 2010. Growth in a time of debt. *The American Economic Review* 100(2), 573-578.

Reinhart, C.M. , Rogoff, K.S., 2013. Financial and sovereign debt crises: Some lessons learned and those forgotten. *IMF Working Paper* no.WP/13/266, International Monetary Fund.

Reinhart, C.M., Rogoff, K.S., Savastano, M.A., 2003. Debt intolerance. *Brookings Papers on Economic Activity* (1), 1-74.

Romer, D., 2012. *Advanced Macroeconomics*. (4th ed.) McGraw-Hill, New York.

Saint-Paul, G., 1992. Fiscal Policy in an Endogenous Growth Model. *The Quarterly Journal of Economics* 107(4), 1243-1259.

Shapiro, M.D., Slemrod, J., 2003. Consumer response to tax rebates. *The American Economic Review* 93(1), 381-396.

Solow, R.M., 1956. A contribution to the theory of economic growth. *The Quarterly Journal of Economics* 70(1), 65-94.

Stiglitz, J.E., Fitoussi, J.-P., Bofinger, P., Esping-Andersen, G., Galbraith, J.K., Grabel, I., 2014. A Call for Policy Change in Europe. *Challenge* 57(4), 5-17.

Teles, V.K., Mussolini, C.C., 2014. Public debt and the limits of fiscal policy to increase economic growth. *European Economic Review* 66(February), 1-15.

Weil, P., 1989. Overlapping families of infinitely-lived agents. *Journal of Public Economics* 38(2), 183-198.

Yakita, A., 2008. Sustainability of public debt, public capital formation, and endogenous growth in an overlapping generations setting. *Journal of Public Economics* 92(3-4), 897-914.

Captions of figures

Figure 1. The ratio of public debt to the GDP in some developed countries (%)

Figure 2. The sensitivity of the crowding-out effect and the resulting output loss in the Eurozone

Figure 3. The marginal effect of public debt and the output loss at a 90% debt-to-GDP ratio as a function of the private saving rate

Figure 4. The marginal effect of public debt and the output loss at a 90% debt-to-GDP ratio as a function of the population growth rate

Supplementary Material for the Reviewers (A): The RCK model

For the restricting assumptions of the model and the meaning of the variables/parameters see the paper.

1. Households

The dynamic optimization problem

$$\max_{i^K,i^B} U=\int_0^\infty \ln(c_t)e^{-(\rho-n)t}dt$$
$$\dot{a}_t=\dot{k}_t+\dot{b}_t=(1-\tau_w)w_t+(1-\tau_A)(r_t^K k_t+r_t^B b_t)-n(k_t+b_t)-c_t$$
$$\dot{k}_t=i_t^K-\delta k_t$$
$$\dot{b}_t=i_t^B$$

, where $i^{\#}$ is the investment per capita into the respective asset type (# = K, B). In the followings, we drop the t time index when no confusion emerges.

The Hamiltonian and the FOCs of the optimization

$$H=\ln(c)e^{-(\rho-n)t}+\upsilon^K(i^K-\delta k)+\upsilon^B i^B=\ln\left[(1-\tau_w)w+(1-\tau_A)(r^K k+r^B b)-n(k+b)-(i^K-\delta k)-i^B\right]e^{-(\rho-n)t}+\upsilon^K(i^K-\delta k)+\upsilon^B i^B$$

, where $\upsilon^{\#}$ is the shadow price of the respective asset type.

$$\frac{\partial H}{\partial i^K}=0=\upsilon^K-\frac{e^{-(\rho-n)t}}{c} \quad , \quad \frac{\partial H}{\partial i^B}=0=\upsilon^B-\frac{e^{-(\rho-n)t}}{c}$$
$$\frac{\partial H}{\partial k}=-\dot{\upsilon}^K=-\upsilon^K\delta+\frac{e^{-(\rho-n)t}}{c}\left((1-\tau_A)r^K-n+\delta\right)=\frac{e^{-(\rho-n)t}}{c}((1-\tau_A)r^K-n)$$
$$\frac{\partial H}{\partial b}=-\dot{\upsilon}^B=\frac{e^{-(\rho-n)t}}{c}((1-\tau_A)r^B-n)$$
$$\frac{\partial H}{\partial i^K},\ \frac{\partial H}{\partial i^B}\ \rightarrow\ \upsilon^K=\upsilon^B=\upsilon \quad \text{and} \quad \frac{\partial H}{\partial k},\ \frac{\partial H}{\partial b}\ \rightarrow\ r^K=r^B=r$$
$$\lim_{t\to\infty}(k_t\upsilon_t^K)=\lim_{t\to\infty}(b_t\upsilon_t^B)=0$$

The equilibrium paths of the consumption and the shadow price

$$\upsilon_t=\upsilon_0 e^{-((1-\tau_A)\bar{r}_t-n)t} \quad \text{, where } \bar{r}_t=\int_0^t r_v dv \Big/ t$$

$$\frac{\dot{c}}{c}=(1-\tau_A)r-\rho \quad \rightarrow c_t=c_0 e^{((1-\tau_A)\bar{r}_t-\rho)t}$$

The intertemporal budget constraint

To derive the intertemporal budget constraint of households, rewrite the flow budget constraint by using the general asset variable (a) instead of the individual asset types:[35]

[35] Note, that the rates of return on the different asset types are equal in equilibrium.

$\dot{a} = (1-\tau_W)w + (1-\tau_A)ra - na - c$, where $a = k + b$.

According to the solution of the flow budget constraint and the transversality condition, the intertemporal budget constraint is

$$\int_0^\infty c_t e^{-((1-\tau_A)\overline{r}_t - n)t} dt = a_0 + \int_0^\infty (1-\tau_W) w_t e^{-((1-\tau_A)\overline{r}_t - n)t} dt = k_0 + b_0 + \overline{W} - \overline{T}_W = b_0 + (\overline{W} + \overline{R}_K) - (\overline{T}_W + \overline{T}_K)$$

, where $\overline{T}_W = \int_0^\infty \tau_W w_t e^{-((1-\tau_A)\overline{r}_t - n)t} dt$, $\overline{T}_K = \int_0^\infty \tau_A r_t k_t e^{-((1-\tau_A)\overline{r}_t - n)t} dt$.

2. Firms

$y = f(\hat{k})$, $r = f'(\hat{k}) - \delta$, $\hat{w} = f(\hat{k}) - \hat{k} f'(\hat{k})$

3. Government

$\dot{B} = rB - (\tau_W W + \tau_A rK + \tau_A rB - G) = (1-\tau_A) rB - \Gamma^*$, where $\Gamma^* = \tau_W W + \tau_A rK - G$ and $G_t / Y_t = \phi$

No Ponzi-game condition: $\lim_{t\to\infty} B_t e^{-(1-\tau_A)\overline{r}_t t} = 0$

$$B_t = B_0 e^{(1-\tau_A)\overline{r}_t t} - e^{(1-\tau_A)\overline{r}_t t} \int_0^t e^{-(1-\tau_A)\overline{r}_s s} \Gamma_s^* ds$$

$$B_0 = b_0 = \int_0^\infty e^{-(1-\tau_A)\overline{r}_t t} [\tau_W W_t + \tau_A r_t K_t - G_t] dt = \overline{T}_W + \overline{T}_K - \overline{G}$$

4. Ricardian Equivalence

$$\int_0^\infty c_t e^{-((1-\tau_A)\overline{r}_t - n)t} dt = (\overline{T}_W + \overline{T}_K - \overline{G}) + (\overline{W} + \overline{R}_K) - (\overline{T}_W + \overline{T}_K) = \boxed{\overline{W} + \overline{R}_K - \overline{G}}$$

5. The dynamics

$$\frac{\dot{\hat{c}}}{\hat{c}} = (1-\tau_A) r - \rho - g$$

$$\dot{\hat{k}}_t = (1-\phi) f(\hat{k}_t) - \hat{c}_t - (n + g + \delta) \hat{k}_t$$

$$\dot{\hat{b}}_t = \left((1-\tau_A) r_t - n - g\right) \hat{b}_t + \phi f(\hat{k}_t) - \tau_w \hat{w}_t - \tau_A r_t \hat{k}_t$$

6. The steady state

$$(1-\tau_A) r^* = \rho + g$$

$$\hat{c}^* = (1-\phi) \hat{y}^* - (n + g + \delta) \hat{k}^*$$

$$\hat{b}^* = \frac{\tau_w \hat{w}^* + \tau_A r^* \hat{a}^* - \phi \hat{y}^*}{r^* - n - g} \to \mu^* = \frac{\hat{b}^*}{\hat{y}^*} = \frac{(\tau_w \hat{w}^* + \tau_A r^* \hat{a}^*)/\hat{y}^* - \phi^*}{r^* - n - g} = \frac{\gamma^*}{r^* - g_Y^*}$$

Supplementary Material for the Reviewers (B): The Blanchard model

For the restricting assumptions of the model and the meaning of the variables/parameters see the paper.

1. Households

The Dynamic Optimization Problem

$$\max_{i^K, i^B} E(U_t) = \int_t^{\infty} \ln(c_{j,v}) e^{-\rho_e (v-t)} dv$$
$$\dot{a}_{j,t} = \dot{k}_{j,t} + \dot{b}_{j,t} = (1-\tau_W) w_{j,t} + (1-\tau_A)(r_{e,t}^K k_{j,t} + r_{e,t}^B b_{j,t}) - c_{j,t}$$
$$\dot{k}_{j,t} = i_{j,t}^K - \delta k_{j,t}$$
$$\dot{b}_{j,t} = i_{j,t}^B$$
$$k_{j,j} = b_{j,j} = 0$$

, where $\rho_e = \rho + p$, $r_e^{\#} = r^{\#} + p$ and $i^{\#}$ is the investment into the respective asset type (# = K, B). In the followings, we drop the *t* time index and/or the *j* cohort index, when no confusion emerges.

The Hamiltonian and the FOCs of the optimization

$$H = \ln(c) e^{-\rho_e t} + \upsilon_K (i^K - \delta k) + \upsilon_B i^B = \ln\left[(1-\tau_w) w + (1-\tau_A)(r_e^K k + r_e^B b) - (i^K - \delta k) - i^B\right] e^{-\rho_e t} + \upsilon_K (i^K - \delta k) + \upsilon_B i^B$$

$$\frac{\partial H}{\partial i^K} = 0 = \upsilon_K - \frac{e^{-\rho_e t}}{c} \ , \ \frac{\partial H}{\partial i^B} = 0 = \upsilon_B - \frac{e^{-\rho_e t}}{c}$$
$$\frac{\partial H}{\partial k} = -\dot{\upsilon}_K = -\upsilon_K \delta + \frac{e^{-\rho_e t}}{c}\left((1-\tau_A) r_e^K + \delta\right) = \frac{e^{-\rho_e t}}{c}(1-\tau_A) r_e^K$$
$$\frac{\partial H}{\partial b} = -\dot{\upsilon}_B = \frac{e^{-\rho_e t}}{c}(1-\tau_A) r_e^B$$
$$\frac{\partial H}{\partial i^K}; \ \frac{\partial H}{\partial i^B} \ \rightarrow \ \upsilon_K = \upsilon_B = \upsilon \ \text{ and } \ \frac{\partial H}{\partial k}; \ \frac{\partial H}{\partial b} \ \rightarrow \ r_e^K = r_e^B = r_e = r + p$$
$$\lim_{t\to\infty}(k_t \upsilon_{K,t}) = \lim_{t\to\infty}(b_t \upsilon_{B,t}) = 0$$

The equilibrium paths of the consumption and the shadow prices

$$\upsilon_{j,t} = \upsilon_{j,j} e^{-\bar{r}_{ne,(t,j)}(t-j)} \ , \ \text{where } \bar{r}_{ne,(t,j)} = \int_j^t r_{ne,v} dv \Big/ (t-j) \ \text{ and } \ r_{ne} = (1-\tau_A) r_e = (1-\tau_A)(r+p)$$

$$\frac{\dot{c}_{t,j}}{c_{t,j}} = r_{ne,t} - \rho_e \quad \rightarrow \ c_{t,j} = c_{j,j} e^{(\bar{r}_{ne,(t,j)} - \rho_e)(t-j)}$$

The intertemporal budget constraint

To derive the intertemporal budget constraint of the households, rewrite the flow budget constraint by using the general asset variable (*a*) instead of the individual asset types:[36]

$$\dot{a} = (1-\tau_W) w + (1-\tau_A) r_e a - c \ .$$

[36] Note, that the rates of returns on the different assets are equal in equilibrium.

According to the solution of the flow budget constraint and the transversality condition, the intertemporal budget constraint is:

$$\int_t^\infty c_v e^{-\bar{r}_{ne,v}(v-t)} dv = a_t + \int_t^\infty (1-\tau_w) w_v e^{-\bar{r}_{ne,v}(v-t)} dv$$

Marginal propensity to consume out of wealth:

$$\int_t^\infty c_v e^{-\bar{r}_{ne,v}(v-t)} dv = \int_t^\infty c_t e^{(\bar{r}_{ne,v}-\rho_e)(v-t)} e^{-\bar{r}_{ne,v}(v-t)} dv = c_t \int_t^\infty e^{-\rho_e (v-t)} dv = c_t \left[\frac{1}{-\rho_e} e^{-\rho_e (v-t)} \right]_t^\infty = \frac{c_t}{\rho + p}$$

$$c_{j,t} = (\rho + p)(a_{j,t} + (1-\tau_w)\bar{w}_t) \quad \text{, where } \bar{w}_t = \int_t^\infty w_v e^{-\bar{r}_{ne,v}(v-t)} dv$$

2. Firms

$$y = f(\hat{k}) \; , \; r = f'(\hat{k}) - \delta \; , \hat{w} = f(\hat{k}) - \hat{k} f'(\hat{k})$$

3. Aggregates

$$C_t = \int_{-\infty}^{t} c_{j,t}(p+n) e^{nj} e^{-p(t-j)} dj \; , \; A_t = \int_{-\infty}^{t} a_{j,t}(p+n) e^{nj} e^{-p(t-j)} dj \; ,$$

$$\bar{W}_t = \int_{-\infty}^{t} \bar{w}_t (p+n) e^{nj} e^{-p(t-j)} dj = \bar{w}_t e^{-pt} \int_{-\infty}^{t} (p+n) e^{(n+p)j} dj = \bar{w}_t e^{nt}$$

3.1 Consumption[37]

$$C_t = (\rho + p)(A_t + (1-\tau_w)\bar{W}_t)$$

$$\dot{C}_t = (\rho + p)(\dot{A}_t + (1-\tau_w)\dot{\bar{W}}_t)$$

$$\dot{A}_t = (r_{ne,t} - p) A_t + (1-\tau_w) w_t e^{nt} - C_t$$

$$\dot{A}_t = \int_{-\infty}^{t} \dot{a}_{j,t}(p+n) e^{nj} e^{-p(t-j)} dj - p \int_{-\infty}^{t} a_{j,t}(p+n) e^{nj} e^{-p(t-j)} dj + \underbrace{a_{t,t}}_{0} (p+n) e^{nt} e^{-p(t-t)}$$

$$\dot{A}_t = \int_{-\infty}^{t} \left[r_{ne,t} a_{j,t} + (1-\tau_w) w_t - c_{j,t} \right] (p+n) e^{nj} e^{-p(t-j)} dj - pA_t$$

$$\dot{A}_t = r_{ne,t} A_t - pA_t - C_t + (1-\tau_w) w_t e^{-pt} \int_{-\infty}^{t} (p+n) e^{(p+n)j} dj$$

$$\dot{\bar{W}}_t = (r_{ne,t} + n)\bar{W}_t - e^{nt} w_t$$

$$\dot{\bar{W}}_t = e^{nt} \left[\int_t^\infty r_{ne,t} w_v e^{-\bar{r}_{ne,v}(v-t)} dv - w_t \right] + n e^{nt} \int_t^\infty w_v e^{-\bar{r}_{ne,v}(v-t)} dv$$

$$\frac{\dot{C}_t}{C_t} = r_{ne,t} + n - \rho - p - (\rho + p)(n + p) \frac{A_t}{C_t}$$

$$\dot{C}_t = (\rho + p)(\overbrace{(r_{ne,t} - p) A_t + (1-\tau_w) w_t e^{nt} - C_t}^{\dot{A}_t} + (1-\tau_w) \overbrace{((n + r_{ne,t})\bar{W}_t - w_t e^{nt})}^{\dot{\bar{W}}_t})$$

$$\dot{C}_t = (\rho + p)((r_{ne,t} - p) A_t - C_t + (1-\tau_w)(n + r_{ne,t})\bar{W}_t) \quad \Big| \quad (1-\tau_w)\bar{W} = \frac{C_t}{(\rho + p)} - A_t$$

$$\dot{C}_t = (\rho + p)(-(n + p) A_t - C_t + (n + r_{ne,t}) \frac{C_t}{(\rho + p)}) \quad \Big| \; : C_t$$

$$\frac{\dot{C}_t}{C_t} = (\rho + p) \left(-(n + p) \frac{A_t}{C_t} - 1 + \frac{n + r_{ne,t}}{\rho + p} \right)$$

[37] Leibniz's rule for differentiation of definite integrals:

$$F = \int_{a(c)}^{b(c)} f(x,c) dx \rightarrow \frac{dF}{dc} = \int_{a(c)}^{b(c)} \frac{df(x,c)}{dc} dx + f(x = b, c) \frac{db(c)}{dc} - f(x = a, c) \frac{da(c)}{dc}$$

$$\frac{\dot{\hat{c}}_t}{\hat{c}_t} = (r_{ne,t} - \rho_e) - g - x\frac{\hat{a}_t}{\hat{c}_t} = (1-\tau_A)(r_t + p) - (\rho + p) - g - (\rho + p)(n+p)\frac{\hat{a}_t}{\hat{c}_t}$$

3.2 Physical Capital

$$K_t = \int_{-\infty}^{t} k_{j,t}(p+n)e^{nj}e^{-p(t-j)}dj$$

$$\hat{k}_t = \frac{1}{EL}\int_{-\infty}^{t} k_{j,t}(p+n)e^{nj}e^{-p(t-j)}dj = e^{-(n+g+p)t}\int_{-\infty}^{t} k_{j,t}(p+n)e^{(n+p)j}dj$$

$$\dot{\hat{k}}_t = -(n+p+g)\underbrace{e^{-(n+g+p)t}\int_{-\infty}^{t} k_{j,t}(p+n)e^{(n+p)j}dj}_{\hat{k}} + \boxed{e^{-(n+g+p)t}\int_{-\infty}^{t} \dot{k}_{j,t}(p+n)e^{(n+p)j}dj}$$

$$\dot{\hat{k}}_t = -(n+p+g)\hat{k}_t + \hat{y}_t - \delta\hat{k}_t + p\hat{k}_t - \hat{c}_t - \phi\hat{y}_t$$

$$\dot{\hat{k}}_t = \hat{y}_t - (n+g+\delta)\hat{k}_t - \hat{c}_t - \phi\hat{y}_t$$

$$e^{-(n+g+p)t}\int_{-\infty}^{t} \dot{k}_{j,t}(p+n)e^{(n+p)j}dj = e^{-(n+g+p)t}\int_{-\infty}^{t}\overbrace{\left[(1-\tau_W)w_t + r_{ne,t}k_{j,t} + r_{ne,t}b_{j,t} - c_{j,t} - \dot{b}_{j,t}\right]}^{\dot{a}_{j,t}-\dot{b}_{j,t}}(p+n)e^{(n+p)j}dj$$

$$= (1-\tau_W)(w_t)e^{-(n+g+p)t}\int_{-\infty}^{t}(p+n)e^{(n+p)j}dj + r_{ne,t}\underbrace{e^{-(n+g+p)t}\int_{-\infty}^{t}(k_{j,t}+b_{j,t})(p+n)e^{(n+p)j}dj}_{e^{-(n+g)t}(K_t+B_t)} -$$

$$\underbrace{e^{-(n+g+p)t}\int_{-\infty}^{t} c_{j,t}(p+n)e^{(n+p)j}dj}_{e^{-(n+g)t}C_t} - \underbrace{e^{-(n+g+p)t}\int_{-\infty}^{t}\dot{b}_{j,t}(p+n)e^{(n+p)j}dj}_{\dot{\hat{b}}_t + \hat{b}_t(g+n+p)}$$

$$= (1-\tau_W)\hat{w}_t + r_{ne,t}(\hat{k}_t + \hat{b}_t) - \hat{c}_t - (g+n+p)\hat{b}_t - \boxed{\dot{\hat{b}}_t}$$

$$= (1-\tau_W)\hat{w}_t + (1-\tau_A)(r_t+p)(\hat{k}_t+\hat{b}_t) - \hat{c}_t - (g+n+p)\hat{b}_t - \left[((r_t - n - g)\hat{b}_t + \phi\hat{y}_t - \tau_W\hat{w}_t - \tau_A(r_t+p)\hat{k} - \tau_A(r_t+p)\hat{b}\right]$$

$$= \hat{w}_t + (r_t+p)\hat{k}_t - \hat{c}_t - \phi\hat{y}_t$$

$$= \hat{y}_t - \delta\hat{k}_t + p\hat{k}_t - \hat{c}_t - \phi\hat{y}_t$$

$$B_t = \int_{-\infty}^{t} b_{j,t}(p+n)e^{nj}e^{-p(t-j)}dj \rightarrow \dot{B}_t = \int_{-\infty}^{t}\dot{b}_{j,t}(p+n)e^{nj}e^{-p(t-j)} - pb_{j,t}(p+n)e^{nj}e^{-p(t-j)}dj = \int_{-\infty}^{t}\dot{b}_{j,t}(p+n)e^{nj}e^{-p(t-j)}dj - pB_t$$

$$\dot{\hat{b}}_t = \frac{\dot{B}_t}{E_tL_t} - \hat{b}_t(g+n) = e^{-(g+n)t}\int_{-\infty}^{t}\dot{b}_{j,t}(p+n)e^{nj}e^{-p(t-j)}dj - p\hat{b}_t - \hat{b}_t(g+n)$$

3.3 Government

$$\dot{B} = rB - \left(\tau_W W + \tau_A (r+p)K + \tau_A (r+p)B - G\right) = ((1-\tau_A)(r+p) - p)B - \Gamma^*,$$

where $\Gamma^* = \tau_W W + \tau_A (r+p)K - G$ and $\phi = G_t / Y_t$

$$\dot{\hat{b}} = \dot{B}/EL - (n+g)b = ((r_{ne} - p) - n - g)\hat{b} + \phi\hat{y} - \tau_W \hat{w} - \tau_A r_e \hat{k}$$

The dynamics of the aggregate consumption implies that in steady state $r_{ne}^* > \rho_e + g \rightarrow r_{ne}^* - p > \rho + g$.

Thus in case of $\rho > n$, the intertemporal budget constraint is realized when the long-run debt-to-GDP ratio is constant.

No Ponzi-game condition: $\lim_{t\to\infty} B_t e^{-(\bar{r}_{ne,t} - p)t} = 0$

$$B_t = B_0 e^{(\bar{r}_{ne,t} - p)t} - e^{(\bar{r}_{ne,t} - p)t} \int_0^t e^{-(\bar{r}_{ne,s} - p)s} \Gamma_s^* ds$$

$$B_0 = \int_0^\infty e^{-(\bar{r}_{ne,t} - p)t} [\tau_W W_t + \tau_A r_{e,t} K_t - G_t] dt$$

4. The dynamics

$$\dot{\hat{k}}_t = (1-\phi) f(\hat{k}_t) - \hat{c}_t - (n+g+\delta)\hat{k}_t$$

$$\frac{\dot{\hat{c}}_t}{\hat{c}_t} = (r_{ne,t} - \rho_e) - g - x\frac{\hat{a}_t}{\hat{c}_t}$$

$$\dot{\hat{b}}_t = (r_{ne,t} - p - n - g)\hat{b}_t + \phi f(\hat{k}_t) - \tau_W \hat{w}_t - \tau_A r_{e,t} \hat{k}_t$$

5. The steady state

$$\hat{c}^* = (1-\phi)\hat{y}^* - (n+g+\delta)\hat{k}^*$$

$$\hat{c}^* = \frac{x\hat{a}^*}{r_{ne}^* - \rho_e - g} = \frac{x\hat{a}^*}{(1-\tau_A)(f'(\hat{k}^*) - \delta + p) - (\rho + p + g)}$$

$$\hat{b}^* = -\frac{\phi f(\hat{k}^*) - \tau_W \hat{w}^* - \tau_A r_e^* \hat{k}^*}{r_{ne}^* - p - n - g}$$

6. The derivation of the formula for the crowding-out effect

$$(1-\phi)\hat{y}^* - (n+g+\delta)\hat{k}^* = \frac{x(\hat{k}^* + \hat{b}^*)}{r_{ne}^* - \rho_e - g} \qquad \Bigg| \; \mu^* = \frac{\hat{b}^*}{\hat{y}^*}$$

$$(1-\phi)\hat{y}^* - (n+g+\delta)\hat{k}^* = \frac{x(\hat{k}^* + \mu^*\hat{y}^*)}{r_{ne}^* - \rho_e - g}$$

$$\mu^* = \left[1 - \phi^* - (n+\delta+g)\frac{\hat{k}^*}{\hat{y}^*}\right]\frac{r_{ne}^* - \rho_e - g}{x} - \frac{\hat{k}^*}{\hat{y}^*}$$

$$\frac{\partial\mu^*}{\partial\hat{k}^*}=\left[-(n+\delta+g)\frac{f(\hat{k}^*)-\hat{k}^* f'(\hat{k}^*)}{f^2(\hat{k}^*)}\right]\frac{r_{ne}^*-(\rho_e+g)}{x}+\left[1-\phi^*-(n+\delta+g)\frac{\hat{k}^*}{f(\hat{k}^*)}\right]\frac{(1-\tau_A)f''(\hat{k}^*)}{x}-\frac{f(\hat{k}^*)-\hat{k}^* f'(\hat{k}^*)}{f^2(\hat{k}^*)}<0$$

$$\boxed{\frac{\partial\mu^*}{\partial\hat{k}^*}\hat{k}^*=\left[-(n+\delta+g)\frac{\hat{w}^*}{\hat{y}^*}\frac{\hat{k}^*}{\hat{y}^*}\right]\frac{\hat{a}^*}{\hat{c}^*}+\left[\frac{\hat{c}^*}{\hat{y}^*}\right]\frac{(1-\tau_A)f''(\hat{k}^*)}{x}\hat{k}^*-\frac{\hat{w}^*}{\hat{y}^*}\frac{\hat{k}^*}{\hat{y}^*}=\frac{1}{\Phi}}$$

7. The derivation of rho

$$\mu^*=\left[1-\phi^*-(n+\delta+g)\frac{\hat{k}^*}{f(\hat{k}^*)}\right]\frac{r_{ne}^*-(g+\rho+p)}{x}-\frac{\hat{k}^*}{f(\hat{k}^*)}$$

$$\frac{\hat{b}^*+\hat{k}^*}{\hat{y}^*}\frac{1}{1-\phi^*-(n+\delta+g)\hat{k}^*/\hat{y}^*}=\frac{r_{ne}^*-(g+\rho+p)}{x}$$

$$\frac{\hat{b}^*+\hat{k}^*}{\hat{y}^*}\frac{1}{1-\phi^*-(n+\delta+g)\hat{k}^*/\hat{y}^*}(p+n)p+\frac{\hat{b}^*+\hat{k}^*}{\hat{y}^*}\frac{1}{1-\phi^*-(n+\delta+g)\hat{k}^*/\hat{y}^*}(p+n)\rho=r_{ne}^*-(g+\rho+p)$$

$$\frac{\hat{b}^*+\hat{k}^*}{\hat{y}^*}\frac{1}{1-\phi^*-(n+\delta+g)\hat{k}^*/\hat{y}^*}(p+n)\rho+\rho=r_{ne}^*-(g+p)-\frac{\hat{b}^*+\hat{k}^*}{\hat{y}^*}\frac{1}{1-\phi^*-(n+\delta+g)\hat{k}^*/\hat{y}^*}(p+n)p$$

$$\rho=\frac{r_{ne}^*-(g+p)-\frac{\hat{b}^*+\hat{k}^*}{\hat{y}^*}\frac{1}{1-\phi^*-(n+\delta+g)\hat{k}^*/\hat{y}^*}(p+n)p}{\frac{\hat{b}^*+\hat{k}^*}{\hat{y}^*}\frac{1}{1-\phi^*-(n+\delta+g)\hat{k}^*/\hat{y}^*}(p+n)+1}=\boxed{\frac{r_{ne}^*-(g+p)-\frac{\hat{b}^*+\hat{k}^*}{\hat{c}^*}(p+n)p}{\frac{\hat{b}^*+\hat{k}^*}{\hat{c}^*}(p+n)+1}}$$

Supplementary Material for the Reviewers (C):

The impact of public debt on s.s. output through distortionary taxation in the RCK model

In order to quantify the impact of public debt on long-run output through distortionary taxation we use the framework of the RCK model presented in section 2 in the paper. The differences compared to section 2 are the followings. First, the tax rates of the wage income and the asset income are set to be equal for mathematical convenience. Second, the production function takes the Cobb-Douglas form. (For the meaning of the variables and parameters, see the paper.)

1. The baseline RCK model (published in the appendix of the paper)

The steady state and the equilibrium of the economy are described partly by the following equations:

(1.1) $\hat{y} = \hat{k}^{\alpha}$

(1.2) $\left(1-\tau^*\right)r^* = \rho + g \;\rightarrow\; r^* = \dfrac{\rho+g}{\left(1-\tau^*\right)}$

(1.3) $r = \alpha\dfrac{\hat{y}}{\hat{k}} - \delta \qquad \rightarrow \; \dfrac{\hat{y}}{\hat{k}} = \dfrac{r+\delta}{\alpha}$

(1.4) $\mu^* = \dfrac{\Gamma^*/Y^*}{r^* - g_Y^*} = \dfrac{\left(\tau^* r^* B^* + \tau^* W^* + \tau^* r^* K^* - G^*\right)/Y^*}{r^* - g - n} = \dfrac{\tau^* r^* \mu^* + \tau^* - \tau^*\delta\hat{k}^*/\hat{y}^* - \phi}{r^* - g - n} \qquad \left|Y = W + rK + \delta K\right.$

Fixed parameters: $\alpha,\ \phi,\ g,\ \rho,\ \delta,\ n$
Policy variable: μ*
Endogenous variables determined by equations (1.1-1.4): $\hat{y}^*,\ \hat{k}^*,\ \tau^*,\ r^*$

<u>The derivation of tau</u>

Substitute eq.(1.2) and (1.3) for *r** and $\hat{k}^*/\hat{y}^*$ in equation (1.4) (asterisks are neglected):

$$\mu = \frac{\tau\mu\dfrac{\rho+g}{(1-\tau)} + \tau - \tau\delta\dfrac{\alpha(1-\tau)}{\rho+g+(1-\tau)\delta} - \phi}{\dfrac{\rho+g}{(1-\tau)} - g - n} \qquad \left| * \frac{\rho+g}{(1-\tau)} - g - n\right.$$

$$\mu\left(\frac{\rho+g}{(1-\tau)} - g - n\right) = \tau\mu\frac{\rho+g}{(1-\tau)} + \tau - \tau\delta\frac{\alpha(1-\tau)}{\rho+g+(1-\tau)\delta} - \phi \qquad \left| -\tau\mu\frac{\rho+g}{(1-\tau)} - \tau + \phi\right.$$

$$\mu\rho - \mu n - \tau + \phi = -\tau\delta\frac{\alpha(1-\tau)}{\rho+g+(1-\tau)\delta} \qquad \left| * \rho + g + (1-\tau)\delta\right.$$

$$\mu\rho^2 - \mu n\rho - \tau\rho + \phi\rho + \mu\rho g - \mu ng - \tau g + \phi g + \mu\rho\delta - \mu n\delta - \tau\delta + \phi\delta - \mu\rho\tau\delta + \mu n\tau\delta + \tau^2\delta - \phi\tau\delta = -\tau\delta\alpha(1-\tau)$$

$$\underbrace{(\delta-\delta\alpha)}_{coeff1}(\tau^*)^2 + \underbrace{(-\rho-g-\delta-\delta\mu^*\rho+\mu^*n\delta-\delta\phi+\delta\alpha)}_{coeff2}\tau^* + \underbrace{(\mu^*\rho^2-\mu^*n\rho+\phi\rho+\mu^*\rho g-\mu^*ng+\phi g+\mu^*\rho\delta-\mu^*n\delta+\phi\delta)}_{coeff3} = 0$$

$$\tau^* = \frac{-coeff_2 \pm \sqrt{(coeff_2)^2 - 4coeff_1 coeff_3}}{2coeff_1}$$

The derivation of *r**: eq.(1.2). The derivation of $\hat{y}^*$ and $\hat{k}^*$: eq.(1.1) and eq.(1.3)

2. The augmented RCK model (not published in the paper)

The economy operates according to the baseline RCK model presented in the paper with the following exceptions: 1. the interest rate on government bonds (r_B) is fixed, 2. the tax rate on the interest income from government bonds (τ_B) is also fixed, 3. human capital (H) and consumption tax (τ_C) are included into the model.[38] The depreciation rates of human capital and physical capital are set to be equal for mathematical convenience: $\delta_H = \delta_K = \delta$. The steady state and the equilibrium of the economy are described partly by the following equations:[39]

(2.1) $\hat{y} = \hat{k}^{\alpha}\hat{h}^{\beta}$, where $\hat{h} = \frac{H}{EL} = \frac{\beta}{\alpha}\hat{k}$, thus $\hat{y} = \hat{k}^{\alpha}\left(\frac{\beta}{\alpha}\hat{k}\right)^{\beta}$,

(2.2) $\left(1-\tau^*\right)r^* = \rho + g \quad \rightarrow \quad r^* = \frac{\rho + g}{\left(1-\tau^*\right)}$,

(2.3) $r = \alpha\frac{\hat{y}}{\hat{k}} - \delta \quad \rightarrow \quad \frac{\hat{y}}{\hat{k}} = \frac{r+\delta}{\alpha}$,

(2.4) $$\mu^* = \frac{\Gamma^*/Y^*}{r_B - g_Y^*} = \frac{\left(\tau_B r_B B^* + \tau^* W^* + \tau^* r^* K^* + \tau^* r^* H^* + \tau_C C^* - G^*\right)/Y^*}{r_B - g - n} \qquad \Big| \; Y = W + r(K+H) + \delta(K+H)$$

$$= \frac{\tau_B r_B \mu^* + \tau^* - \tau^*\delta\hat{k}^*/\hat{y}^* - \tau^*\delta\hat{h}^*/\hat{y}^* + \tau_C\hat{c}^*/\hat{y}^* - \phi}{r_B - g - n} \qquad \Big| \; \hat{h} = \frac{\beta}{\alpha}\hat{k}$$

$$= \frac{\tau_B r_B \mu^* + \tau^* - \tau^*\delta((\alpha+\beta)/\alpha)\hat{k}^*/\hat{y}^* + \tau_C\hat{c}^*/\hat{y}^* - \phi}{r_B - g - n}$$

(2.5) $\frac{\hat{c}^*}{\hat{y}^*} = (1-\phi) - (n+g+\delta)\frac{\alpha+\beta}{\alpha}\frac{\hat{k}^*}{\hat{y}^*}$.

Fixed parameters: $\alpha,\ \beta,\ \rho,\ g,\ \delta,\ \phi,\ r_B,\ \tau_B,\ \tau_C,\ n$

Policy variable: μ^*

Variables determined by equations (2.1-2.5): $\tau^*,\ r^*,\ \hat{y}^*,\ \hat{k}^*,\ \hat{h}^*,\ \hat{c}^*$.

<u>The derivation of tau</u>

Substitute eq.(2.5) and eq.(2.3) for $\hat{c}^*/\hat{y}^*$ and $\hat{k}^*/\hat{y}^*$ in equation (2.4) respectively (asterisks are neglected):

$$\mu = \frac{\tau_B r_B \mu + \tau - \tau\delta\frac{\alpha+\beta}{\alpha}\frac{\hat{k}}{\hat{y}} + \tau_C(1-\phi) - \tau_C(n+g+\delta)\frac{\alpha+\beta}{\alpha}\frac{\hat{k}}{\hat{y}} - \phi}{r_B - g - n}$$

[38] The full RCK model with human capital and consumption tax is provided upon request.

[39] In equilibrium, the rates of return on human capital (r_H) and physical capital (r_K) are equal. Thus $\alpha\frac{\hat{k}^{\alpha}\hat{h}^{\beta}}{\hat{k}} - \delta = \beta\frac{\hat{k}^{\alpha}\hat{h}^{\beta}}{\hat{h}} - \delta$.
Equation (2.5) is derived according to the equation of motion of the broad capital: $\dot{\hat{k}} + \dot{\hat{h}} = (1-\phi)\hat{y} - \hat{c} - (n+g+\delta)(\hat{k}+\hat{h})$.

$$\mu = \frac{\tau_B r_B \mu + \tau - (\tau\delta + \tau_C (n+g+\delta)) \frac{\alpha+\beta}{\alpha} \frac{\alpha(1-\tau)}{\rho+g+(1-\tau)\delta} + \tau_C (1-\phi) - \phi}{r_B - g - n}$$

$$\mu(r_B - g - n) - \tau_B r_B \mu - \tau + \phi - \tau_C (1-\phi) = -(\tau\delta + \tau_C (n+g+\delta)) \frac{(\alpha+\beta)(1-\tau)}{\rho+g+(1-\tau)\delta} \qquad | \; *(\rho+g+\delta-\tau\delta)$$

$$\mu(r_B - g - n)(\rho+g+\delta) - \mu(r_B - g - n)\tau\delta - \tau(\rho+g+\delta) + \tau\delta\tau + (\phi - \tau_C(1-\phi) - \tau_B r_B \mu)(\rho+g+\delta) - (\phi - \tau_C(1-\phi) - \tau_B r_B \mu)\tau\delta =$$
$$= -\tau\delta(\alpha+\beta) + \tau^2\delta(\alpha+\beta) - \tau_C(n+g+\delta)(\alpha+\beta) + \tau_C(n+g+\delta)(\alpha+\beta)\tau$$

$$\overbrace{\left[\delta - \delta(\alpha+\beta)\right]}^{coeff\,1}(\tau^*)^2 + \overbrace{\left[-\mu^*(r_B - g - n)\delta - (\rho+g+\delta) - (\phi - \tau_C(1-\phi) - \tau_B r_B \mu^*)\delta + \delta(\alpha+\beta) - \tau_C(n+g+\delta)(\alpha+\beta)\right]}^{coeff\,2}\tau^* +$$
$$+ \underbrace{\mu^*(r_B - g - n)(\rho+g+\delta) + (\phi - \tau_C(1-\phi) - \tau_B r_B \mu^*)(\rho+g+\delta) + \tau_C(n+g+\delta)(\alpha+\beta)}_{coeff\,3} = 0$$

$$\tau^* = \frac{-coeff_2 \pm \sqrt{(coeff_2)^2 - 4coeff_1 coeff_3}}{2coeff_1}$$